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Marcus Hagedorn, Ashok Kaul and Tim Mennel

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Marcus Hagedorn[†], Ashok Kaul[‡], and Tim Mennel[§]

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Abstract

We ask whether offering a menu of unemployment insurance contracts is welfare-improving in a heterogeneous population. We adopt a repeated moral hazard framework as in Shavell/Weiss (1979), supplemented by unobserved heterogeneity about agents' job opportunities. Our main theoretical contribution is an analytical characterization of the sets of jointly feasible entitlements that renders an efficient computation of these sets feasible. Our main economic result is that optimal contracts for "bad" searchers tend to be upward-sloping due to an adverse selection effect. This is in contrast to the well-known optimal decreasing time profile of benefits in pure moral hazard environments that continue to be optimal for "good" searchers in our model.

Keywords: Unemployment Insurance, Recursive Contracts, Adverse Selection, Repeated Moral Hazard

JEL classification: J65, J64, D82, C61, E61

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*The views expressed herein are solely those of the authors and do not necessarily reflect the views of the European Central Bank.

†European Central Bank, Kaiserstr. 29, 60311 Frankfurt, Germany; and IZA, Bonn, Germany; email: marcus.hagedorn@ecb.int

‡Institute for Empirical Research in Economics (IEW), University of Zurich, CH-8006 Zurich, Switzerland; and Gutenberg University Mainz, Welderweg 4, 55099 Mainz, Germany; email: ashok.kaul@uni-mainz.de.

§Corresponding author, Zentrum für Europäische Wirtschaftsforschung (ZEW), Postfach 103443 D-68034 Mannheim, Germany; email: mennel@zew.de

1 Introduction

People flowing into unemployment are heterogeneous with respect to their chances of finding a new job. These differences include different innate ability, general education, human capital accumulated on the the last job and their unemployment history. Also, chances of reemployment generally vary over different segments of the labor market. Typically, workers searching for a job are better informed about their job opportunities than a government agency in charge of providing unemployment insurance (UI). This asymmetric information may prevail due to unobservable characteristics or because it is too costly to elicit information. The literature on optimal unemployment insurance generally neglects this unobserved heterogeneity [Karni (1999) provides a comprehensive overview].[1] It mainly focuses on the trade-off between setting incentives to search and insurance aspects of providing benefits to a representative agent.[2] This is remarkable since self-selection from a menu of contracts is common practice when dealing with heterogeneous populations in the design of health insurance or car insurance markets.

In this paper, we ask whether offering a *menu of unemployment insurance contracts* is welfare improving and what the optimal contracts look like. We adopt a multi-period contract theoretical framework, introduced by Shavell and Weiss (1979) and later refined and simulated by Hopenhayn and Nicolini (1997). We extend their representative agent framework to encompass heterogeneity in the job search technology, i.e., we introduce types of agents that differ with respect to their probability of finding a job when controlling for search effort. We compute the optimal menu of UI contracts offered by a UI agency from which each agent selects his preferred contract.

As Hopenhayn and Nicolini (1997), our paper technically builds on recursive solutions of repeated games and principal-agent problems as analyzed in the papers by Spear and Srivastava (1987), Thomas and Worrall (1990), Abreu, Pearce and Stacchetti (1990), Atkeson and Lucas (1992) and Chang (1998). However, our combination of adverse selection with *repeated* moral hazard has not been studied yet. Furthermore, concerning the adverse selection problem, agents' types, which parameterize their chances of finding a job, are drawn only once but affect their chances in all future periods. This permanent heterogeneity in our model implies that the first period is distinct from the following periods. Both the adverse selection incentive constraints and the entitlement constraints in the principal's problem have to hold

[1] Exceptions are Mortensen (1983) and Wang and Williamson (2002). For a discussion see below.

[2] Atkinson and Micklewright (1991) and Meyer (1995) survey the empirical literature that documents disincentive effects from providing UI.

only in the first period whereas moral hazard incentive constraints hold in every period. Although we assume full commitment, there is no "natural" way to state such a problem recursively. Additional state variables that restrict the planner's choices have to be added, similar to Fernandes and Phelan (2000).

In the numerical solution procedure we face an unresolved difficulty (in the general case) in the recursive contracts literature building on the Abreu et al. (1990) methodology, namely the efficient calculation of the sets of "sustainable outcomes".[3] Our main theorem provides a precise *analytical* characterization of the set of sustainable outcomes in our setup. This characterization renders a numerical implementation feasible. In particular, we prove certain topological properties of the set of sustainable outcomes that ensure an efficient computation that considerably improves in terms of accuracy upon previous algorithms [compare e.g. section 8 of Chang (1998)].

The paper answers the following questions. First, under which circumstances will it be optimal to offer only one UI contract to all agents - a situation that resembles real world UI schemes in most countries? Second, are all optimal UI contracts decreasing, as in the representative-agent setup [see Fredriksson and Holmlund (2006)]? We can show analytically (under fairly standard conditions) that in the optimum separating contracts should be offered. We then investigate the shape of optimal contracts numerically in a framework where two types of agents (good and bad) face different hazard rates of finding a job (given a certain search effort). We show that the contract for the good searcher has a decreasing benefit profile, as the one he would be offered in a pure moral hazard environment. In contrast, the contract of the bad searcher will tend to have an upward-sloping benefit profile.[4] The reason for this can be understood by recalling the simpler setup when agents only differ in their hidden exogenous job finding probabilities (so that there is adverse selection, but no moral hazard). In that case, good searchers tend to enjoy a flat profile (full consumption smoothing)) and receive an information rent over their promised utility. The bad searcher faces a higher risk of longer unemployment duration and will consequently discount future unemployment benefit payments less than the good type. An increasing profile partially insures bad searchers against unemployment without giving good searchers much of an incentive to claim they are bad searchers

[3]This terminology was introduced by Abreu et al. (1990). It describes the sets of contracts that are implementable taking into account future choices which again have to take into account initial choices and so on and so forth.

[4]Werning (2002) and Kocherlakota (2004) consider optimal unemployment insurance with unobservable savings (in contrast savings are observable in our paper). They find that optimal unemployment benefits are not necessarily decreasing with unemployment duration.

since good searchers value later payments of benefits less. This is the basic consideration for self-selection of the agents. In our more general case with moral hazard and adverse selection, the slope of the bad type's contract is ambiguous since the moral hazard consideration induces a negative slope whereas the adverse selection consideration induces an increase in the slope (relative to the pure moral hazard contract). The slope of the good type's contract is unambiguously negative since moral hazard requires setting search incentives (as in Shavell and Weiss (1979)) relative to the full consumption smoothing benchmark. We also provide a numerical comparative-statics analysis of changes in various parameters and give a detailed intuition of our results.

In the optimal-UI literature the issue of adverse selection has been raised first by Mortensen (1983) who applies the seminal Rothschild and Stiglitz (1976) paper to UI. His analysis is, however, static and does not include search incentives. Wang and Williamson (2002) present a numerical welfare analysis of the effect of (full and partial) experience rating on optimal UI in a dynamic economy with moral hazard and heterogeneous agents. Their framework is different from ours in that their model includes endogenous work effort as well as precautionary saving but they assume that there is the same UI scheme for all agents. In contrast in our model agents choose from a set of different UI contracts offered by the principal. Hopenhayn and Nicolini (2001) address the issue of heterogeneity of agents in a two period model of UI similar to their earlier work. They assume, however, that the agents' type is observable and contractible. Lentz (2005) estimates a heterogeneous agent search model on Danish data. He also considers optimal UI but he does not compute the optimal benefit profile over the unemployment spell and there is no adverse selection problem. Joseph and Weitzenblum (2003) find that taking into account transitional dynamics in the presence of precautionary savings substantially affects the optimal replacement ratio. Heterogeneity in their model is different from this paper, as it refers to different asset holdings. They also do not consider the optimal UI profile.

The paper is organized as follows. In section 2 the model is introduced and our assumptions are motivated. In section 3 the analytical results are presented. In section 4 the model is solved numerically; we also present an extensive comparative statics exercise. Section 5 discusses various possible extensions of our model. Proofs are given in the appendix.

2 Model

Our framework for analyzing UI is a dynamic principal-agent model. The principal represents the government (or UI agency) providing social insurance.

The agents' problem: The agents are unemployed workers searching for a job. There is a continuum of agents, modeled by the unit interval. The agents are of two types, differing in their opportunities of finding a new job. The differences between the two types of agents in their search technology are expressed by agent i's probability of remaining unemployed $p_i(a)$, which is a function of the search effort a he exerts (we will sharpen the notion of heterogeneity formally below). The agents have private information about their types. The fraction of agents of type B ("bad searcher") q and of type G ("good searcher") $1-q$ are common knowledge. Except for the difference in search technologies (as formalized below), we assume that agents are identical. In particular, we assume that both agents are equally risk-averse, and enjoy the same utility $u(.)$ from consumption. In addition, we assume that once an agent starts work , he will keep his job with a fixed wage rate w until his death in period T.[5] We can thus calculate an *employed agent's total expected lifetime utility in period* t as:

$$W_t = \sum_{l=t}^{T} \beta^{l-t} u(w),$$

where β is the discount factor for both type of agents and the principal.[6]

Unemployed agents receive possibly time-varying UI benefits. We denote by b_t the benefit received in period t. Given any benefit scheme $\{b_1, ..., b_T\}$, the agent chooses his search effort a_t in any period in which he is unemployed. In his decision to increase his search effort he faces a trade-off between increasing search costs on the one hand and an increasing probability of reemployment on the other. The total lifetime utility he expects when remaining unemployed is the key variable determining this decision. We denote by V_{t+1}^i an *unemployed agent's total expected lifetime utility in period* $t+1$, where $i = B, G$. Moreover, in the sequel we will

[5]Technically, the assumption that both agents receive the same wage is in no way important for our analysis; it is introduced for the sake of simplicity. In the solution to our model we could keep track of the impact of different wages for B and G on the search incentives; qualitatively, the results would not change.

[6]The restriction to a finite time horizon T is a computational requirement (see below). We could, as in Wang and Williamson (2002), without changing the results of this paper, relax this assumption and assume that agents live forever but UI is received for T periods only.

denote by $z_t = u(b_t)$ the utility value of consuming benefit b_t. Thus we can state agent i's problem ($i = B, G$) in period t recursively by:

$$V_t^i = \max_a z_t - a + \beta[p_i(a)V_{t+1}^i + (1 - p_i(a))W_{t+1}], \tag{1}$$

where we assumed that effort enters utility linearly. Recall that W_{t+1} is an employed agent's expected lifetime utility where employment starts in $t+1$. We denote by $\hat{a}_t^i$ the decision of agent i:

$$\hat{a}_t^i = \mathrm{argmax}_a\, z_t - a + \beta[p_i(a)V_{t+1}^i + (1 - p_i(a))W_{t+1}]. \tag{2}$$

The principal will have to take into account the decisions by the agents when designing contracts for them. These constraints are known as *moral hazard incentive constraints* (MH-IC).

The principal's problem: The principal's objective is to minimize the cost of providing a certain "level" of insurance by the design of (a menu of) optimal contracts for the agents. Hereby the current cost function $c(.)$ is the inverse of the agent's utility function: $c(z_i) \equiv u^{-1}(z_i) = b_i$. Future costs are discounted with the discount factor β. We will call $\{z_1^i, ..., z_T^i\}$ a *contract designed for agent* i (with $i = B, G$). We will often use the terms *contract* b and *contract* g to denote the contracts designed for agents B and G respectively. Agents choose one contract from the offered contracts in period 0. We assume that the principal can fully commit to the contract promised in period zero. The following property of $c(.)$ is implied by monotonicity and strict concavity of an agent's utility function $u(.)$, i.e. risk aversion.

Condition 2.1 *The cost function* $c(.)$ *is increasing and is strictly convex.*

In his minimization problem, the principal has to take into consideration the different reemployment probabilities of the agents. Furthermore, he has to take into account the following constraints: First, as mentioned above, he has to take into consideration the agents' decision problem, i.e. the moral hazard incentive constraints. Second, the principal has to guarantee incentive compatibility due to adverse selection, i.e. he has to ensure that each agent chooses the contract designed for him. Third, he has to respect the entitlement $\underline{V}$ (i.e. total expected lifetime utility) that the contracts should at least guarantee to agents B and G respectively. This entitlement can be interpreted as the "level of insurance" the principal is willing to guarantee. Any value of the entitlement $\underline{V}$ can be mapped one-to-one to a "certainty equivalent replacement rate", i.e. to a percentage of the wage w which is consumed every period and which provides lifetime utility of exactly $\underline{V}$. This interpretation will be used to calibrate reasonable values of $\underline{V}$ later on.

The principal's problem can thus be stated as:

$$\min_{\{z_1^B,\ldots,z_T^B\},\{z_1^G,\ldots,z_T^G\}} q[c(z_1^B)+\beta p_B(\hat{a}_1^B)[c(z_2^B)+\beta p_B(\hat{a}_2^B)[c(z_3^B)+\ldots]\ldots]]+$$
$$(1-q)[c(z_1^G)+\beta p_G(\hat{a}_1^G)[c(z_2^G)+\beta p_G(\hat{a}_2^G)[c(z_3^G)+\ldots]\ldots]]$$

subject to the *entitlement constraints* (EC)

$$V_1^{b,B} \geq \underline{V}, \tag{3}$$
$$V_1^{g,G} \geq \underline{V}, \tag{4}$$

and the *adverse selection incentive constraints* (AS-IC)

$$V_1^{b,B} \geq V_1^{g,B}, \tag{5}$$
$$V_1^{g,G} \geq V_1^{b,G}. \tag{6}$$

The hat on the a's describing the choices of effort of the agents, $\hat{a}$, indicates that the principal respects the (MH-IC). We abbreviate the principal's problem by (ASUI). In the formulation of the (ASUI), $V_t^{j,i}$ denotes *total expected lifetime utility in period t for the unemployed agent i ($i = B, G$) if he selects contract j ($j = b, g$).* The superscript j indicates for which agent the contract is designed, i.e. contract b is designed for agent B and contract g for agent G. *A priori* both agents can of course choose either contract before period 1, although the AS-IC constraints ensure that they will in fact choose the contract designed for them. The entitlement constraints guarantee that the chosen contract gives the promised utility. Both the $V_t^{j,i}$s and the $\hat{a}_t^i$s can be calculated recursively from the array of equations 1 and 2 given a contract.

Remark 2.2 *If agents are homogeneous in their search costs, i.e. they all have the same $p_i(a)$, then the setup is identical to the one considered by Shavell and Weiss (1979) (except for the finite time dimension).*

To make our problem interesting, the initial entitlements to total expected lifetime utility have to be below the total lifetime utility from work. If the entitlements $V_t^{i,j}$ are higher than W_t for any period posterior to one in that the unemployed agents exert search effort, the efforts would necessarily be zero, and thus the probability of remaining unemployed p_i would be 1.

Condition 2.3 *The utility entitlement of the unemployed agent is below the one guaranteed by lifetime work:* $\underline{V} < W_1$.

We will see in the sequel that this condition guarantees that all optimal $V_t^{i,j}$ are smaller than W_t.

Formalization of the agents' heterogeneity: Now we formalize in detail the idea that agents differ in their reemployment probabilities. To develop the model formally, we first make some standard technical assumptions on the $p_i(.)$ functions.

Condition 2.4 *The probability of remaining unemployed* $p_i(a)$ *of agent* i*:*

1. *Smoothness:* $p_i(a) \in C^\infty(\mathbf{R})$.
2. *Monotonicity and strict convexity:* $p_i'(a) < 0$, $p_i''(a) > 0$.
3. *Boundary conditions:* $p_i(0) = 1$, $\lim_{a\to\infty} p_i(a) = 0$.
4. *Inada conditions:* $\lim_{a\to 0} p_i'(a) = -\infty$, $\lim_{a\to\infty} p_i'(a) = 0$.

Condition 2.4 ensures that the agents' problem (1) always has a unique interior solution that can be characterized by a first order condition.

The basic difference between the two types of agents in their search technology is expressed by agent i's probability of remaining unemployed $p_i(a)$, which is a function of the search effort a he exerts:

Condition 2.5 *Given the same effort, type B has a higher probability of remaining unemployed than type G:* $p_B(a) > p_G(a)$.

Condition 2.6 (Spence-Mirrlees property)

$$\frac{\partial V_t^B(z_1, ..., z_T)}{\partial z_{t+s}} - \frac{\partial V_t^G(z_1, ..., z_T)}{\partial z_{t+s}} > 0 \tag{7}$$

for all t *with* $1 \le t \le T$ *and* s *with* $1 \le s \le T - t$.

Condition 2.6, which is standard in classical contract theory, sharpens the basic concept of a bad or good searcher. It implies that - faced with the *same* contract - the bad searcher may exert a higher search effort than the good searcher under this contract, but his effort will not be so high that his chance of finding a job exceeds the good searcher's chance:

Lemma 2.7 *Condition 2.6 holds if and only if agents facing the same contract* $\{z_1, ..., z_T\}$, *in equilibrium exert efforts such that* $p_B(a_t^B) > p_G(a_t^G)$.

Conditions 2.4 to 2.6 will be used in all results that follow. For our first result - on the optimality of separating the types - we need one more assumption on $p_i(.)$. As we mentioned above, Condition 2.4 ensures that the agent's choice of effort from (1) can be characterized by the following first order condition:

$$p_i'(a_t^i) = \frac{1}{\beta(V_{t+1}^i - W_{t+1})}. \tag{8}$$

Equation (8) establishes a one-to-one and smooth relation between V_t^i and a_t^i. We can therefore define the following function for the next-to-last period:

$$\pi_i(z_T) \equiv p_i(a_{T-1}^i) = p_i\left((p_i')^{-1}\left(\frac{1}{\beta(z_T - u(w))}\right)\right)$$

So $\pi_i(z_T)$ is agent i's probability of remaining unemployed when facing benefit utility z_T in the last period. It is clear that π_i is increasing. We formulate a condition on the elasticity of π_i with respect to z_T, which - of course - implicitly puts restrictions on the choice of $p_i(.)$.

Condition 2.8

1. *The marginal probability of remaining unemployed* $\frac{\partial \pi_i(z)}{\partial z}$ *of agent* i *facing promised utility* z *is greater for agent G than for agent B:*

$$\frac{\partial \pi_G(z)}{\partial z} > \frac{\partial \pi_B(z)}{\partial z}$$

2. *If the marginal probabilities* $\frac{\partial \pi_i(z)}{\partial z}$ *of agent G and B are equal for two utility values* z^G *and* z^B, *then the utility of G,* z^G, *must be smaller than the utility of B,* z^B*:*

$$\frac{\partial \pi_G(z^G)}{\partial z} = \frac{\partial \pi_B(z^B)}{\partial z} \;\;\Rightarrow\;\; z^G < z^B.$$

What is the economic content of Condition 2.8? The first part says that agent G reacts more strongly to a change in the promise z than agent B. In other words: The probability of finding a new job depends more critically on the UI benefit promise in the case of agent G than in the case of agent B. Note that here we compare agents G and B that face the same contract. The second part of Condition 2.8 says that whenever the reaction is equal, then agent G must face a *lower* promise than agent B. Summarizing Condition 2.8 we can say that the "incentive sensitivity" of agent G is higher than that of agent B.

What role do Conditions 2.6 to 2.8 play in our analysis? The model presented in this paper incorporates two different paradigms, hidden information and (repeated) hidden action. Condition 2.6 is the typical technical assumption in hidden information models. Condition 2.8 is a condition that ensures in our setup a feature of (pure) repeated hidden action that has been analyzed numerically in Pavoni (forthcoming) and discussed in Hopenhayn and Nicolini (2001): In the full information case, the decline of the UI benefits over time is sharper for better searchers. Loosely speaking, Conditions 2.6 and 2.8 ensure that our model exhibits the "standard" behavior of a pure hidden information model and a pure hidden action model. We will see in Section 4 that both conditions will be met in the functional specification of our simulation.

3 Theoretical Results

In this section we develop a characterization of the solution to the principal's problem. First, we will however turn to a standard question in contract theory.

Pooling is not optimal: The first question we ask is whether and under what circumstances it is actually optimal to offer two contracts in order to screen the agents. The answer gives a first indication that it may indeed be relevant to consider the cost-saving potential of a differentiated UI.

Proposition 3.1 *There exists a solution to the Principal's Problem (ASUI). If Condition 2.8 holds, any solution is separating.*

What makes the theorem very appealing from a more applied perspective is the fact that our numerical implementation with Constant Relative Risk Aversion (CRRA) utility shows that the good searcher typically reacts more sensitively (in the sense of Condition 2.8) to the search incentives than the bad searcher. Considering this numerical result as robust, we could claim that the UI agency has a definite potential for cost-saving by switching from offering only one to offering two UI contracts.

But what should the optimal contracts look like? The answer to this question is not immediately obvious: We can neither apply the Shavell and Weiss (1979) approach directly, since we should expect the influence of hidden information on the optimal design of the contracts, nor can we apply standard solutions of adverse selection models that do not incorporate repeated hidden action. Moreover, we cannot hope to find a direct recursive formulation of the problem, because both the adverse selection incentive constraints and the entitlement constraints in the principal's problem (ASUI) have to hold *only* in the first period.

The solution of the model: In the sequel, we develop a characterization of the solution to the principal's problem (ASUI) in Propositions 3.2, 3.3 and 3.7. The strategy is as follows: We first give a quasi-recursive formulation of the principal's problem (ASUI), i.e. a formulation that splits up (ASUI) into an adverse selection problem formulated in terms of each agent's total lifetime utilities and two cost functions that are themselves solutions to recursive problems. The philosophy is to look first at each contract *separately*, ignoring for a moment the issue of self-selection. Proposition 3.2 gives a recursive formulation of the problem of finding a cost-minimizing contract that provides agents B and G with two arbitrarily specified levels of ex ante lifetime utility (*a pair of first-period entitlements*) *if* both of them choose this contract. The recursive formulations summarize the cost minimization for each contract in a compact way. In this formulation the pairs of entitlements (one pair for each contract) and their evolution over time serve as state variables of the problem (cf. Hopenhayn and Nicolini (2001)). We then merge the two separately solved cost minimization problems and state the original adverse selection problem faced by the principal as a four-dimensional minimization problem in the (two pairs of) entitlements of the *first* period.

The question left open by Proposition 3.2 is which pairs of entitlements are actually jointly feasible under a *given* contract (still ignoring the issue of self-selection). The answer to this question has to take into account the choices of effort by the agents induced by the contract under consideration as well as the laws of motion for the entitlements. Proposition 3.3 gives a precise theoretical and numerically useful description of the correspondence Γ_t, mapping pairs of entitlements of the agents today (V_t^B, V_t^G) to jointly feasible policy options $(z_t, V_{t+1}^B, V_{t+1}^G)$, i.e. the benefit today and the entitlements for tomorrow. This proposition is the main theoretical result of the paper and serves as the cornerstone of our recursive numerical implementation. We can now calculate the cost of a given contract that provides agents with any feasible pairs of *first period* entitlements.

Finally, we can further simplify the adverse selection problem stated in Proposition 3.2. Proposition 3.7 shows that the entitlement constraint of G must be slack, and that the entitlement constraint of B and the adverse selection incentive constraint of G must be binding at the solution. This reduces the dimension of the problem from four to two which allows us to solve the problem with high numerical accuracy.

We begin the characterization of the solution by a reformulation of the principal's problem (ASUI). The adverse selection problem is stated in terms of recursive subproblems for contract b and contract g, i.e. the contract designed for agents B and G respectively. As usual in adverse selection problems, we anticipate that only agent B will choose contract b in the end and thus stochastically discount costs at his rate. For the time being, the lifetime (or first period) entitlements of contract b for agents B and G, $V^{b,B}$ and $V^{b,G}$, are taken as given. Their optimal values for given entitlement constraints from the original problem will be calculated in Proposition 3.7 below. The recursive formulation of the contracts takes the form of a (finite-dimensional) Bellman equation: The principal minimizes the costs of paying out a benefit worth z_t (in utility units) today and promising entitlements $V_{t+1}^{b,B}$ and $V_{t+1}^{b,G}$ for tomorrow. In doing so, he has to observe the entitlements of B and G today, $V_t^{b,B}$ and $V_t^{b,G}$, which serve as state variables of the problem. A law of motion connects the state and the choice variables. We denote the set of possible choices $(z_t^b, V_{t+1}^{b,B}, V_{t+1}^{b,G})$ in state $(V_t^{b,B}, V_t^{b,G})$ by $\Gamma_t(V_t^{b,B}, V_t^{b,G})$. This correspondence will be characterized later in Proposition 3.3. Moreover, the choices of effort a_t^i from the agents' problem facing the promised entitlement V_{t+1}^i for tomorrow, i.e. the MH-IC, are taken as given by the principal. The recursive formulation is completed by two boundary conditions: The first period entitlements $V_1^{b,B}$ and $V_1^{b,G}$ of course have to equal the promised ex ante lifetime utilities $V^{b,B}$ and $V^{b,G}$ respectively. In the last period, the entitlements $V_T^{b,B}$ and $V_T^{b,G}$ have to take the value of the last period benefit z_T^b. To see this, recall that we consider agents who chose the same contract, namely contract b. So both receive a benefit of z_T in the last period. But in the last period, unlike all other periods, there is no way of splitting the promise for that period into a benefit in that period and a promise one period later, because there *is no period later*.

The following proposition gives a quasi-recursive characterization of the principal's problem (ASUI).

Proposition 3.2 (Quasi-recursive formulation of ASUI) *The problem (ASUI) can be reformulated in the following way:*

$$
\begin{aligned}
&\min_{V^{b,B},V^{b,G},V^{g,B},V^{g,G}} qC_1^b(V^{b,B},V^{b,G}) + (1-q)C_1^g(V^{g,B},V^{g,G}) && (9)\\
s.t.\quad & \Gamma_1(V^{b,B},V^{b,G}) \neq \emptyset && (10)\\
& \Gamma_1(V^{g,B},V^{g,G}) \neq \emptyset && (11)\\
& V^{b,B} \geq V^{g,B} && (12)\\
& V^{g,G} \geq V^{b,G} && (13)\\
& V^{b,B} \geq \underline{V} && (14)\\
& V^{g,G} \geq \underline{V}. && (15)
\end{aligned}
$$

In this formulation we use cost functions $C_1^B(V^{b,B},V^{b,G})$ *and* $C_1^G(V^{g,B},V^{g,G})$ *in the lifetime utilities* $V^{i,j}$ *that contract* i *guarantees to agent* j *(with* $i \in \{b,g\}$ *and* $j \in \{B,G\}$*). We also use the correspondence* $\Gamma_t(V_t^{i,B},V_t^{i,G})$ *that maps the lifetime utilities in period* t*,* $(V_t^{i,B},V_t^{i,G})$ *onto a set of jointly feasible choice variables* $(z_t,V_{t+1}^{i,B},V_{t+1}^{i,G})$*. The cost functions have the following recursive form (in order to facilitate the presentation, we concentrate on contract* b*, dropping the index* b *from* $V_t^{b,j}$*):*

$$
C_t^b(V_t^B,V_t^G) = \min_{\{z_t^b,V_{t+1}^B,V_{t+1}^G\}\in\Gamma_t(V_t^B,V_t^G)} c(z_t^b) + \beta p_B(a^B)C_{t+1}^b(V_{t+1}^B,V_{t+1}^G) \qquad (16)
$$

subject to:
Law of motion for contract b *(LOM)*

$$
\begin{aligned}
z_t^b - a^B + \beta[p_B(a^B)V_{t+1}^B + (1-p_B(a^B))W_{t+1}] &= V_t^B\\
z_t^b - a^G + \beta[p_G(a^G)V_{t+1}^G + (1-p_G(a^G))W_{t+1}] &= V_t^{b,G},
\end{aligned}
$$

choice of effort in contract b *(MH-IC)*

$$
\begin{aligned}
a^B &= \mathrm{argmax}_a\, z_t^b - a + \beta[p_B(a)V_{t+1}^B + (1-p_B(a))W_{t+1}]\\
a^G &= \mathrm{argmax}_a\, z_t^b - a + \beta[p_G(a)V_{t+1}^G + (1-p_G(a))W_{t+1}],
\end{aligned}
$$

as well as the boundary conditions

$$
\begin{aligned}
V_1^B &= V^{b,B} && (17)\\
V_1^G &= V^{b,G} && (18)\\
V_T^B &= z_T^b && (19)\\
V_T^G &= z_T^b. && (20)
\end{aligned}
$$

Proof. See appendix. ■

In order to make use of Proposition 3.2, we actually have to be able to calculate the correspondence $\Gamma_t(.,.)$ as precisely as possible. This is particularly important for any numerical application of the recursive formulation. The following proposition gives a characterization of the theoretical properties of Γ_t that are indispensable for a satisfactory approach to calculate Γ_t numerically. For technical reasons we distinguish between the case where the utility z_t from consuming the UI benefit is bounded from below, and the case where it is not. We will discuss this and other issues after stating the proposition.[7]

Proposition 3.3 (Characterization of Γ_t) *The following formulas characterize the correspondence $\Gamma_t(V_t^B, V_t^G)$, which produces all feasible policy options for given entitlements V_t^B and V_t^G in period t. Note that we allow for a lower bound $\underline{z}$ on the utility value of the benefit z_t. The reason for this will be explained in Remark 3.4.*

1. *Be $t \leq T$ and let $z_t \geq \underline{z}$ (where $\underline{z}$ may take the value $-\infty$). Then there exists a lower bound $\underline{V}^G$, so that for $\underline{V}^G \leq V_t^G \leq W_t$:*

   $$\Gamma_t(V_t^B, V_t^G) = \begin{cases} \{z_t(a), V_{t+1}^B(a), V_{t+1}^G(a)\}_{a \in [\underline{a}, \overline{a}](V_t^B, V_t^G)} & : \quad V_t^B \in [\underline{V}_t^B(V_t^G), \overline{V}_t^B(V_t^G)] \\ \emptyset & : \quad else \end{cases} \tag{21}$$

   The jointly feasible values $z_t(a)$, $V_{t+1}^B(a)$ and $V_{t+1}^G(a)$ are differentiable functions.
   The boundary functions $\underline{a}(V_t^B, V_t^G)$, $\overline{a}(V_t^B, V_t^G)$ and $\underline{V}_t^B(V_t^G)$, $\overline{V}_t^B(V_t^G)$ are continuous. They can depend on the value of $\underline{z}$ (when $z_t \geq \underline{z}$ is binding).
   For V_t^G below $\underline{V}^G$, the correspondence $\Gamma_t(V_t^B, V_t^G)$ is the empty set.

[7] The notation $a \in [\underline{a}, \overline{a}](V_t^B, V_t^G)$ means that the boundaries $\underline{a}$ and $\overline{a}$ are functions $\underline{a}(V_t^B, V_t^G)$ and $\overline{a}(V_t^B, V_t^G)$.

2. *In period $T-1$ the correspondence $\Gamma_{T-1}(V^B_{T-1}, V^G_{T-1})$ takes the same form as in 1 with only one possible parameter value a, i.e. only one choice $\{z_{T-1}(a), V^B_T(a), V^G_T(a)\}$.*

In the recursive formulation of contract b (Proposition 3.2), the principal has to choose a three-dimensional vector $(z_t, V^B_{t+1}, V^G_{t+1})$ in every period. Proposition 3.3 shows that feasibility restricts the principal's choice set to a one-dimensional path, smoothly parameterized through a. Furthermore the support of Γ is a convex set, a property that is crucial for the numerical implementation.

Two further technical remarks on Proposition can be made: 3.3.

Remark 3.4 *The upper bound W_t on V^G_t is artificial: Of course the principal can ensure lifetime utilities above the value of secure lifetime income from work. However, this cannot be optimal, since it reduces the search effort to zero, and in view of Condition 2.3, we exclude lifetime utilities above W_t from our considerations.*

Remark 3.5 *The lower bound on z_t in Proposition 3.3 is introduced for technical reasons: Some utility functions map onto the real line $\mathbf{R}$, while some only onto the half-line $\mathbf{R}_+$. Constant Absolute Risk Aversion (CARA) are an example of the former kind and CRRA utility functions with risk aversion smaller than one are an example of the latter kind.*

With Proposition 3.3 at hand, we can precisely define the notion of feasibility in our model: A pair of entitlements (V^B_t, V^G_t) is called *jointly feasible* if the set $\Gamma_t(V^B_t, V^G_t)$ is non-empty. Note that the correspondence Γ_t maps into the values of jointly feasible (z_t, V^B_{t+1}, V^G_{t+1}) (see the last line of Proposition 3.2), means $\Gamma_{t+1}(V^B_{t+1}, V^G_{t+1})$ is automatically non-empty.

Let us now be more specific as to why this proposition is so important for our purposes. Models including repeated moral hazard, like ours, have been discussed in the framework of a strand of literature building on Spear and Srivastava (1987), Thomas and Worrall (1990), Abreu et al. (1990) [APS], Atkeson and Lucas (1992) and Chang (1998). The following remark relates Proposition 3.3 to this literature.

Remark 3.6 *The sets of jointly feasible entitlements are the finite-dimensional analogue of the set of sequential equilibrium payoffs (of the agents' game) in the infinite-dimensional framework of APS or the set of sustainable outcomes in the (again infinite-dimensional) framework of Chang (1998).*

By introducing entitlements, marginal utilities or sequential equilibrium outcomes as state variables - instead of "intuitive" state variables - we inevitably run into the difficulty of precisely defining the sets of possible values these state variables can take. APS and Chang (1998) characterize these sets as the largest fixed point of a set operator. Moreover, they show that the fixed point can be obtained by a fixed-point iteration of sets. This is theoretically sound. However, it does not provide an entirely satisfactory description of the sets nor the definite algorithm to calculate them numerically, in particular if the state space is more than one-dimensional. In fact, the numerical determination of these sets may be a tricky issue in simulations of models building on these methods. In Proposition 3.3 we give - for our model - a satisfactory description of the sets of the state variables. The boundaries of the sets are continuous functions. In particular, the sets are compact, connected and contractible.[8] In the next section we will point out that this is crucial for the numerical implementation of our solution. Moreover, Proposition 3.3 states that the principal's choice problem in a given period is essentially one-dimensional (in the sense that the correspondence describes a smooth one-dimensional path in the three-dimensional real space, with this path being parameterized in a).[9]

After this methodological digression, we return to the solution of the principal's problem (ASUI). Proposition 3.2 states (ASUI) as a four-dimensional minimization problem, a still rather complex problem from a numerical viewpoint. However, as in the case of standard adverse selection problems (see for example the book by Laffont and Martimort (2002), Chapter 2), we are able to show that agent B's entitlement constraint and agent G's adverse selection incentive constraint must be binding, and that agent G's entitlement constraint must be slack at the solution. And so we finally characterize the solution to the principal's problem (ASUI) as follows:

Proposition 3.7 (Solution of the principal's problem) *For values of $\underline{V}$ that are not too low, solutions to the principal's problem are solutions to the simplified problem:*

$$\begin{aligned} &\min_{V^{g,B},V^{g,G}} qC_1^B(\underline{V}, V^{g,G}) + (1-q)C_1^G(V^{g,B}, V^{g,G}) \\ &s.t. \quad \underline{V} \geq V^{g,B} \end{aligned}$$

The qualification "not too low" is needed to avoid corner solutions. In the simulation we found that all values of $\underline{V}$ corresponding to reasonable levels of insurance

[8]Here, "contractible" is a term from algebraic topology. Intuitively, a topological space is contractible if it contains no holes. Formally, all closed loops in the space are homotopic to a single point.

[9]Except for the next-to-last period, where there is only one choice left.

were high enough (compare the proof in the appendix and footnote 11 in subsection 4.1).

Two corollaries: Two corollaries ensue from Proposition 3.7. The setup considered by Shavell and Weiss (1979) will be our benchmark, i.e. a set-up where the principal knows the type of the agent and sets the benefits to give optimal search incentives. We will call this the *pure moral hazard environment*. About the contract for type B we learn:

Corollary 3.8 *Type B receives the minimal entitlement utility $\underline{V}$. His contract is distorted with respect to the optimal contract in a pure moral hazard environment such that its value $V^{b,G}$ for type G is reduced.*

In the case of the contract for type G, we deduce:

Corollary 3.9

1. *Type G receives an information rent, i.e. the utility $V^{g,G}$ that his contract provides him with is greater than $\underline{V}$.*

2. *If the adverse selection incentive constraint of the bad searcher (12) is slack at the solution, the good searcher's contract is identical to the optimal contract in the pure moral hazard environment (given the level of entitlement $V^{g,G}$).*

In our numerical simulations, we found that the adverse selection incentive constraint of the bad searcher (12) was *always* slack.
We thus recover the rent extraction/efficiency trade-off from a simple adverse selection model without moral hazard (cf. Chapter 2 of Laffont and Martimort (2002)). The bad searcher's contract is distorted and his search effort is not efficient, whereas the good searcher's contract is optimal.

The corollaries provide a first step towards a description of how an unemployment agency should design optimal UI contracts. We will discuss in more detail how optimal contracts look after the simulation in the next section.

4 Numerical Analysis

4.1 Computational Strategy

The simulation closely follows Propositions 3.2, 3.3 and 3.7 in Section 2. The first part of the simulation calculates the correspondence Γ_t by backward induction. As in Proposition 3.3, we use the entitlement of agent G, V_t^G, as a parameter for the upper and lower bound on the entitlement V_t^B for agent B. We introduce a grid on V_t^G and then calculate the bounds on V_t^B by a bracketing procedure.

More precisely, for a given tuple (V_t^B, V_t^G) of state variables, we check whether the corresponding path of choice variables $(z_t(a), V_{t+1}^B(a), V_{t+1}^G(a))$ (compare the proof of Proposition 3.3, Appendix C) intersects the set of jointly feasible values $(z_t, V_{t+1}^B, V_{t+1}^G)$ as determined in the previous induction step. As defined in the previous section, by "jointly feasible" we refer to tuples (V_{t+1}^B, V_{t+1}^G) such that $\Gamma_{t+1}(V_{t+1}^B, V_{t+1}^G)$ is non-empty; in the case of z_t, we only have to check whether it is above the lower bound $\underline{z}$. From Proposition 3.3, we know that for each V_t^G (within the limits of feasible entitlements for G) there exits a $\underline{V}_t^B(V_t^G)$ and $\overline{V}_t^B(V_t^G)$ that limits the range of V_t^B jointly feasible with V_t^G. Since for every V_t^G the set of jointly feasible V_t^B is *one* interval, we can "encircle" $\underline{V}_t^B$ (and, separately, $\overline{V}_t^B$) by values of V_t^B above and below and then calculate the bound by a (highly precise) bracketing procedure. Proposition 3.3 thus guarantees that our algorithm calculates a characterization of the set of jointly feasible entitlements (V_t^B, V_t^G) by stating that this set (V_{t+1}^B, V_{t+1}^G) is compact and connected.

As we have pointed out in the preceding section (compare Remark 3.6 and the following discussion), the virtue of Proposition 3.3 lies in a more "precise" characterization of the sets of jointly feasible entitlements (Changs's set of sustainable outcomes). It is exactly here in the numerical algorithm where this characterization becomes useful: The description of the set of sustainable outcomes as a fixed point of a set operator by Abreu et al. (1990) is *mathematically* precise, but poses a serious precision problem in numerical applications with more than one state variable (compare the discussion in Section 8 of Chang (1998)). As an illustration, Figure 1 shows the support of the correspondence $\Gamma_4(V_4^B, V_4^G)$ in the calibration of our model, which is described in the following subsection.

The second part of our numerical procedure uses the recursive formulation in Proposition 3.2. It calculates a numerical approximation of the cost functions $C_t^i(V_t^B, V_t^G)$ based on a solution of the minimization problems in the backward induction of the principal.

More precisely, we cover the domain of C_t^i (i.e. the set of jointly feasible entitlements (V_t^B, V_t^G) for which $\Gamma_t(V_t^B, V_t^G)$ is non-empty) by a large grid. For each tuple (V_t^B, V_t^G) in the grid ("states of the world"), we solve the minimization problem along the path of choice variables $(z_t(a), V_{t+1}^B(a), V_{t+1}^G(a))$, i.e. we solve it in a. Ignoring the exact value of the limits $\underline{a}$ and $\overline{a}$, we use a bracketing procedure in which we allocate an extremely high cost to a values delivering choice variables outside the set of feasible values. Note that we make use of the characterization of Γ_t in two ways: First, we rely on the fact that the set of jointly feasible entitlements (V_t^B, V_t^G) is compact and contractible. [10] Second, we exploit the reduction of the number of choice variables from three $(z_t, V_{t+1}^B, V_{t+1}^G)$ to one a.

Finally, each cost function is then approximated as a linear combination of complete Chebychev polynomials by regression (for this standard procedure, see Judd (1998), Chapters 6.4, 6.12 and 12.8).

In the third part, the approximated cost functions C_1^i are combined in the objective function of the principal's problem. According to proposition 3.7, we have to solve a two-dimensional minimization problem.[11]

After an initial grid search, the solution is calculated using a Nelder-Mead multidimensional minimization procedure.

4.2 Calibration of the Model

In our calibration, we work with a monthly interval. Therefore we set the discount rate to $\beta = 0.995$, which corresponds to an annual discount rate of 0.95. The overall timeframe is a year, i.e. the number of periods is set to $T = 12$. As for the probability function, we choose $p_i(a) = 1 - \sqrt{1 - \exp(-\theta_i a)}$, where θ_i remains to be determined. We use CRRA utility functions $u(b) = \frac{b^{1-\gamma}}{1-\gamma}$, as is common practice in the UI literature. The corresponding cost function $c(.) = u^{-1}(.)$ meets the convexity condition (2.1):

$$c(z) = z^{\alpha},$$

where $\alpha = \frac{1}{1-\gamma} \geq 1$. In addition, Conditions 2.4, 2.6 and 2.8 are fulfilled by the probability function $p_i(a)$.[12] So in particular, the prerequisites of Theorem 3.1 hold in our numerical setup.

In the benchmark case, following Hopenhayn and Nicolini (1997), we set $\alpha = 2$ (i.e. risk aversion $\gamma = \frac{1}{2}$), which corresponds to intermediate risk aversion on behalf of the agents.

[10] If it were not contractible there could be holes in the set of jointly feasible state vectors (V_t^B, V_t^G), and we would have to split up the minimization path into several parts - a tedious and difficult task.

[11] There is one point to take care of, though: In order to apply Proposition 3.7, we have to ensure that the minimal entitlement $\underline{V}$ is so high that $z_1 > \underline{z}$ (compare the proof in Appendix C) We ran alternative minimization routines for low values of $\underline{V}$, showing that the assertion holds.

[12] Our probability function $p_i(a)$ has a slightly more intricate functional form than the one used by Hopenhayn and Nicolini (1997) ($p_{HN}(a) = 1 - \exp(-ra)$). We have chosen it because the latter does not fulfill the Inada condition (cf. Condition 2.4).

The wage is normalized to be $w = 100$, so that unemployment benefits become equal to replacement rates. The lower bound on the UI benefits is set to $\underline{z} = 0$, the lowest possible value taken by CRRA utility functions with $0 < \gamma < 1$. In our simulation - consistent with the proofs of the propositions - we have normalized utility from consuming the wage to zero, i.e. all expected lifetime utilities are negative.

Finally, we choose the parameters $\theta_1 = 0.007$ and $\theta_2 = 0.017$ to match the escape rates from unemployment under the current US unemployment insurance system (compare Meyer (1990)). For type B, the bad searcher, this is then 22.7% per month; for type G, the good searcher, it is 35.8% per month. As a comparison, Hopenhayn and Nicolini (1997) find an average weekly escape rate of 10% for the US, which corresponds to a monthly escape rate of 34.4%. The share of the bad searchers in the unemployment pool, q, is set equal to 0.5. Other choices of parameters will be discussed in the next section, where we give a comparative statics analysis.

4.3 Results

In this section, we discuss the optimal contracts in our environment in comparison to the Shavell-Weiss (SW) pure moral hazard case as well as in comparison with the first-best, i.e. the case where there is no information problem at all. In all cases, the information rent of the good searcher is taken into account to make the contracts comparable.

Figure 2 shows the optimal UI contracts under adverse selection for our choice of parameters described above. The entitlement bound is $\underline{V} = -25$ (we consider different values below), the lifetime (12 periods) utility of a bad searcher who becomes unemployed and receives an replacement rate of 68.4% for six months. In terms of consumption equivalents, an unemployed bad searcher would be as well off if he consumed (a constant) 61.76% of the market wage $w = 100$ for the rest of his life.

The solution is to separate types since Theorem 3.1 applies. The incentive constraint of agent B, equation 12, is slack, and the contract g - designed for the good searcher G according to Corollary 3.9 - is falling and identical to the one considered by Shavell and Weiss (1979). Contract b is rather flat. As well as agent B's effort choices it is distorted relative to agent B's SW contract (which is falling, pure moral hazard case, Figure 4), as stated in Corollary 3.8. Of course, the first-best contracts, where the principal knows the type of the agents and can set their effort choices, are flat (full insurance, Figure 6) and their levels are determined by the initial entitlements of agents B and G.

Figures 3, 5 and 7 show the equilibrium job-finding probabilities of agents B and G, corresponding to the contracts with adverse selection (Figure 3), the SW contracts (Figure 5) and the first best contracts (Figure 7) respectively. As for a comparison of our adverse selection model and second the SW contract, we know that the good searcher gets the same contract with or without adverse selection, and so his reemployment probabilities do not differ. This is markedly different for Agent B, whose reemployment probability is on average 32.7% higher for the SW contract. Furthermore, the profile of reemployment probabilities is (slightly) hump-shaped with adverse selection (whereas it is of course decreasing without adverse selection). Recall that it is the SW contract which sets the search incentives optimally. The relatively flat profile for B in our adverse selection case induces a lower search effort in comparison to the pure moral hazard case (but helps to save information rent). Note that in the first-best contract, the effort choices and reemployment probabilities fall over time. This can be explained by the finite time horizon: The closer the death of the agent, the less valuable is the exit from unemployment, and so the principal prefers not to impose a strong (and costly) effort. Observe, however, that the reemployment probabilities in the first-best case are considerably higher than in the two other cases.

More generally, the size of the distortions and the precise shape of contract b are determined by two effects:

1. A *moral hazard effect* (MH), arising, as in the case of type G, from the agents' search problem,

2. An *adverse selection effect* (AS), arising from the principal's wish to lower the value of the contract for type G (compare Corollary 3.8) in order to separate the types.

We know that in the pure MH environment, benefit schemes are falling. What would agent B's contract look like in a *pure AS environment*? For the latter, we consider a setup where the type of an agent is still hidden information, but the probabilities of remaining unemployed of types B and G are fixed constants $p_B > p_G$. This is then a typical adverse selection problem, as discussed in Chapter 2 of Laffont and Martimort (2002). Due to the assumption of full commitment, the dynamics of the contracts are rather simple. Now, as in the case of the full problem (see Corollary 3.9), agent G receives an information rent, and, given the entitlement $V^{g,G}$, his contract is the first-best contract. In the pure AS case, this means his consumption is fully smoothed. As in the full problem the following Spence-Mirrlees property holds:

$$\frac{\partial V_t^B}{\partial z_{t+s}} - \frac{\partial V_t^G}{\partial z_{t+s}} > 0 \qquad \forall s \geq 1.$$

Table 1: **Information Rents Obtained by Agent G**

$\underline{V}$	Certainty Equivalent	$q = 0.2$	$q = 0.5$	$q = 0.8$
-35	49.03	10.8 %	18.1 %	22.2 %
-30	55.21	7.3 %	13.9 %	17.0 %
-20	68.67	3.1 %	7.2 %	9.9 %

Rents are expressed as a percentage increase over the minimal entitlement

Table 2: **Elasticities of Unemployment Probability w.r.t. Benefit Level for Agent** B

$\underline{V}$	Certainty Equivalent	$\theta_B = 0.004$	$\theta_B = 0.007$	$\theta_B = 0.010$
-35	49.03	0.0070	0.0116	0.0167
-30	55.21	0.0075	0.0131	0.0188
-20	68.67	0.0083	0.0146	0.0209

Therefore, in order to separate the two types, the contract for agent B has to show an *increasing* benefit scheme.[13] In our benchmark calibration both effects cancel each other out and thus contract b is flat.

In the rest of this section we explore the dependence of the optimal contracts on different choices of parameters. In particular, we discuss the influence of different parameters on the relative weight of the MH and the AS effects. For all parameter values the incentive constraint of agent B is slack, and contract g is determined through moral hazard only and is therefore falling (for a detailed numerical discussion of the comparative statics of the contract in a pure moral hazard environment, see Pavoni (forthcoming)). However, the level of entitlement for G (and so, in particular, his information rent) and contract b vary with the parameters of our model.

The entitlement bound $\underline{V}$: For the good searcher, a decrease in entitlement shifts his contract g uniformly downwards. For contract b, Figures 2, 8 and 9

[13] A formal derivation of the solution to the principal's problem in a pure adverse selection environment is available upon request.

(entitlement bounds of $\underline{V} = -25$, $\underline{V} = -20$ and $\underline{V} = -30$) show that the profile is downward-sloping for high entitlement bounds $\underline{V}$ and upward sloping for low entitlement bounds $\underline{V}$. The reason is that the MH effect prevails for high entitlement bounds $\underline{V}$ and the AS effect dominates for low entitlement bounds $\underline{V}$. At the same time, according to Table 1, the information rent for agent G increases as $\underline{V}$ falls.

These results are due to the following considerations of the principal. At low levels of $\underline{V}$, providing strong incentives does not save many costs since transfers are already low. Thus the MH effect is less important and the AS effect dominates. At high levels of $\underline{V}$ on the other hand, strong incentives are important as they save a large amount of money. Therefore the MH effect dominates.

There is another mechanism that reinforces the MH effect as $\underline{V}$ increases (but is quantitatively less important). As can be seen from Table 2, the elasticity of the probability of remaining unemployed $\sigma_{\pi}^{i}(z)$ with respect to UI benefits increases in the entitlement level. This means that at higher levels of utility, a reduction in future benefits has a greater effect on the search effort and thus the reemployment chances of the unemployed agent. Hence MH considerations for the bad searcher matter relatively more than AS effects at higher levels of utility.
A flatter contract b (because of a lower $\underline{V}$) is more attractive to the risk-averse agent G than a steeper contract b (at a high level of $\underline{V}$). Since agents are optimally separated in equilibrium, the principal has to grant G a higher information rent from his contract g when B gets a flatter profile because of a lower $\underline{V}$.

There are three remaining parameters with intuitive comparative statics.[14]

The share of type B agents q: As the share of the bad searcher's q increases, contract b gradually shifts from a contract dominated by the AS effect to one dominated by the MH effect. The contract for the good searcher is uniformly shifted upwards. Moreover, from Table 1 we can see that his information rent also rises.

If the proportion of type B agents is high, then reducing the costs arising from their contract b dominates the principal's problem and the issue of paying an information rent to type G agents becomes less important. In order to keep the costs of contract b low, the distortion of contract b away from its first best is reduced. That is, the MH effect dominates and G's information rent increases.

If on the other hand the proportion of type B agents is low, keeping information rents for G low is the primary objective of the principal. Setting incentives for agent B is then less of a concern, i.e. the MH effect is of minor importance.

[14]The working paper version contains numerous figures that graphically show the effects discussed subsequently.

Table 3: **Information Rents Obtained by Agent G**

$\underline{V}$	Certainty Equivalent	$\theta_B = 0.004$	$\theta_B = 0.007$	$\theta_B = 0.010$
-35	49.03	20.6 %	18.1 %	14.2 %
-30	55.21	14.3 %	13.9 %	10.9 %
-20	68.67	6.6 %	7.2 %	6.2 %

(rents expressed as percentage increase over minimal entitlement)

Agent B's unemployment probability parameter θ_B: The parameter θ_B determines the search capacity of agent B. The higher it is, the lower is his probability of remaining unemployed, given the same search effort a. What happens if we keep $\theta_G = 0.017$ constant and gradually increase θ_B?

Two extreme cases can help us to understand the economic forces at work. If θ_B is zero, setting incentives is ineffective and there is no MH effect at all. As θ_B increases and approaches θ_G, the two contracts b and g become increasingly similar and eventually identical to the optimal contract in a pure MH setup. Thus, the AS effect vanishes. Obviously, the shift in the principal's focus from AS to MH is monotone in θ_B. Therefore, for low values of θ_B the AS effect dominates, whereas for high values the MH effect is dominant.

Risk aversion $\frac{\alpha-1}{\alpha}$: The exponent in the cost function α determines the risk aversion of both agents. The coefficient of relative risk aversion is $\frac{\alpha-1}{\alpha}$.

A higher level of risk aversion has two effects. First, one immediate consequence of a higher risk aversion is that optimal contracts become flatter since agents have a stronger preference for intertemporal consumption-smoothing (numbers available upon request). The second effect, namely that G receives a higher information rent as risk aversion increases, is a consequence of this. To see this, recall that a flatter profile for agent B implies that his contract b becomes more attractive for agent G. Since it is optimal to separate agents in equilibrium, the principal has to grant G a higher information rent from the contract g designed for him when agent B gets a flatter profile in order to separate the agents.

5 Extensions

In this section we want to discuss two possible extensions of our model. A first extension concerns the (minimal) entitlement $\underline{V}$ for the unemployed, which could be type-dependent. Second, we show how to integrate UI taxes which are paid after reemployment (following Hopenhayn and Nicolini (1997)).

Type dependent entitlement constraint: We show in this subsection that we can compute the optimal contract without our assumption that both types (good and bad searchers) have the same entitlement constraint. To organize this discussion we draw on subsection 3.3.1 of Laffont and Martimort (2002), where the optimal contract is characterized in a static environment and for arbitrary entitlement constraints.[15] Laffont and Martimort (2002) show that the solution can be described through five cases, characterized by different combinations of participation and incentive constraints that are binding at the optimal solution.

Case 1 is identical to the one we have found here. The entitlement constraints of the bad searcher and the incentive constraint of the good searcher are binding.

In Cases 2-5, the minimal entitlement of agent G, $\underline{V}_G$, is greater than $V^{g,G}$ and thus, by Corollary 3.9, greater than the entitlement of agent B, $\underline{V}_B$.[16] As the wedge between $\underline{V}_B$ and $\underline{V}_G$ widens, the entitlement constraint of B and finally the incentive constraint of agent B become binding. Moreover, the incentive constraint of agent G and finally the entitlement constraint of agent B become slack. The characterization of Laffont and Martimort (2002) shows that this description is correct.

In Case 2, both entitlement constraints and the good searcher's incentive constraint are binding. In Case 3, both entitlement constraints and no incentive constraints are binding. In Case 4, both entitlement constraints and type B's incentive constraint are binding. In Case 5, type G's entitlement and type B's incentive constraint are binding.

A characterization of which constraints are binding or slack in terms of primitives seems untractable in our dynamic model due to the highly non-linear dependence of the entitlements on the parameters of the problem. Instead, we conduct a numerical analysis. We pick the value of $\underline{V}_B = -30$ and choose a higher number for $\underline{V}_G > V^{g,G}$

[15]The entitlement constraints in our model technically correspond to participation constraints in the adverse selection models analyzed in classical contract theory. The case of type-dependent reservation utility in participation constraints has been widely discussed in that context, the most general work being the article by Jullien (2000).

[16]Since the entitlement constraint of agent G is slack in our optimal solution, any value for agent G's entitlement lower than $V^{g,G}$ does not change the solution.

(for the case of $\underline{V}_B = \underline{V}_G = -30$ (adverse selection), compare Figure 8). We then compute for each case, the (constrained) optimal solution under the assumption that Case 1-5 are true. For example for Case 5, we minimize the principal's cost under the assumption that type G's entitlement and type B's incentive constraint are binding. This results in five cost levels for the principal, one for each case. The optimal contract is then the one with the lowest cost level.

We illustrate this procedure for two different levels of $\underline{V}_G$, -15 and -22. For $\underline{V}_G = -22$ (which corresponds to 65.86% of certainty equivalent consumption), we find that both entitlement constraints and no incentive constraints are binding (case 3). The optimal contract for the good and the bad searcher is then equal to the contract with moral hazard only (the SW contract, see subsection 4.3).[17] The case is depicted in Figure 10. For the higher value $\underline{V}_G = -15$ (75.96% of certainty equivalent consumption), type G's entitlement and type B's incentive constraint are binding (Case 5) (see Figure 11).

Taxes on wages: Shavell and Weiss (1979) analyze the optimal allocation of UI benefits with one representative agent; Hopenhayn and Nicolini (1997) add taxes on labor income after reemployment to this analysis. They make the simplifying assumption that the tax rate is fixed for the rest of their life the moment agents have gained reemployment. In our model we follow Shavell and Weiss (1979), but it is theoretically straightforward to extend it to the framework of Hopenhayn and Nicolini (1997). By taxing (or subsidizing) labor income the principal can completely control the agent's consumption when he is employed. The additional ability of the principal to tax the agent can thus be captured by the introduction of entitlements while employed, W_t, which take the role of additional state variables, and a value function for transfers that corresponds to the cost function in the recursive formulation of the UI contract in the SW model. In our model we would have to introduce two additional state variables for each contract i, $W^{i,B}$ and $W^{i,G}$. These state variables are the entitlement utility of an employed agent under contract i. Since the wage rate is known, the tax rate can be computed once W^i is known and vice versa.

Although it is easy to write down this extension, there is a numerical problem. In Proposition 3.7 we showed that the principal's problem involves only two state variables instead of four variables as in the original problem. This reduction of the state space is crucial since with more state variables, the curse of dimensionality

[17] The replacement rate is higher than 100% in some periods. This reflects the fact that our model does not incorporate the effects of UI on work effort and its impact on employment.

kicks in. This is what would happen now if we allowed wages to be taxed.

In the recursive formulation of contract b (and g likewise) given by Proposition 3.2, the additional state variables $W^{i,B}$ and $W^{i,G}$ enter into the cost functions of the principal and - as choice variables - into the minimization problem in the backward induction. While the values of the transfers after the agent's reemployment are easily calculated, the approximation of the cost function poses a serious numerical problem. This is because it takes now four state variables (for every contract, two from the original problem plus the two additional state variables $W^{i,B}$ and $W^{i,G}$). The curse of dimensionality then becomes a serious problem (see Chapter 6 of Judd (1998)). The other steps of the computation, the minimization problem of the principal (Proposition 3.7) and the choice problem (characterized as one-dimensional in Proposition 3.3) do not pose a serious problem. Only the dimension of the state space of the cost function can prevent computation.

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Appendix

A Proofs

Proof of Lemma 2.7

Proof. Firstly, we prove that $p_B(a_t^B) > p_G(a_t^G)$ ensues from the Spence-Mirrlees Property 7. We calculate

$$\frac{\partial V_t^i}{\partial z_{t+1}} = \beta p_i(a_t^i)\frac{\partial V_{t+1}^i}{\partial z_{t+1}} = \beta p_i(a_t^i),$$

where we use the Envelope Theorem. The assertion now follows immediately.

Secondly, assume that $p_B(a_t^B) > p_G(a_t^G)$ holds. In the case where $s = 1$, the Spence-Mirrlees property follows from what we have shown above. So let $s > 1$. Then

$$\frac{\partial V_t^B}{\partial z_{t+s}} - \frac{\partial V_t^G}{\partial z_{t+s}} = \beta p_B \frac{\partial V_{t+1}^B}{\partial z_{t+s}} - \beta p_G \frac{\partial V_{t+1}^G}{\partial z_{t+s}},$$

where we have used the Envelope Theorem once more. The assertion follows by induction. ■

Proof of Theorem 3.1

Proof. It is clear that there is a solution to the principal's problem. Let assumption 2.8 hold. We prove the second assertion by contradiction: Assume that the principal's problem is solved by one contract p, $\{z_1^P, ..., z_{T-1}^P, z_T^P\}$, for both agents, this generates a total expected utility of V_t^B and V_t^G in period t for agents B and G respectively.

We have a look at the "first best" solutions for the last two periods that generate the same utilities V_{T-1}^B and V_{T-1}^G as p. As a "first best" solution, we refer to the solution of the pure moral hazard problem as considered by Shavell and Weiss (1979), i.e. the problem of guaranteeing agent i a utility of V_{T-1}^i at the lowest cost. This is stated as follows:

$$\begin{aligned} &\min\nolimits_{z_{T-1}^i, z_T^i} c(z_{T-1}^i) + \beta p_i(a^i)c(z_T^i) \\ s.t. \quad V_{T-1}^i &= z_{T-1}^i - a^i + \beta[p(a^i)z_T^i + (1-p(a^i))u(w)] \\ 1 &= \beta p_i'(a^i)[z_T^i - u(w)] \end{aligned}$$

This is the two-period cost minimization problem (the principal's problem in this framework) in the case of agent i, subject to the promise-keeping constraint and the first order condition of the agent's problem, determining the choice of effort a^i. Plugging the entitlement constraint into the objective function and making use of the Enveloppe Theorem, we calculate the following first order conditions for the principal with respect to z_T (we abbreviate $p_i = p_i(a^i)$):

$$c'(z_{T-1}^B) = -\frac{(p_B')^3}{p_B p_B''} c(z_T^B) + c'(z_T^B) \tag{22}$$

$$c'(z_{T-1}^G) = -\frac{(p_G')^3}{p_G p_G''} c(z_T^B) + c'(z_T^B). \tag{23}$$

The factor of the cost function on the right-hand side (RHS) is

$$-\frac{(p_i')^3}{p_i p_i''} = \frac{1}{\pi_i(z_T)} \frac{\partial \pi_i(z_T)}{\partial z_T},$$

and so we see that the RHS is identical to the relative expected marginal cost. By Condition 2.8, part 1, we know that the factor of the cost function is higher for agent G than for agent B for a given z_T. By its second part we know that this has to hold in equilibrium, too, and so the RHS is greater for agent G than for agent B.[18] We may therefore deduce that the SW contract of agent B is flatter than its counterpart for agent G, where we define "flatter" in the following sense:

$$\frac{z_{T-1}^G}{z_T^G} > \frac{z_{T-1}^B}{z_T^B}.$$

In the following we will discuss the last two periods of the pooling contract only and show that it cannot be optimal to offer it to both agents. We will refer to the first-best solutions as SW contracts.

First, suppose that the pooling contract p is flatter than the SW contract of agent G. Then the principal can offer p and a second contract g' that is identical to contract p except for the last two periods, where z_{T-1}^P and z_T^P are substituted by z_{T-1}^G and z_{T-1}^G from the SW contract. This is incentive-compatible: Agent G is indifferent between p and g' by construction. Suppose that agent B (weakly) preferred g' over p. Then for period 1 to $T-2$, he can exert the same effort a_1^g to a_{T-2}^g (i.e. that he chooses in the case of contract g') when facing contract p, and thus the stochastically discounted utility from the benefits z_1 to z_{T-2} is identical for both contracts. In the last two periods, in contrast, agent B - exerting effort optimally - gains a higher utility from the flatter contract p than from contract g' because of the Spence-Mirrlees property (cf. Lemma 2.7). So agent B cannot prefer g' over p. Offering the two contracts p and g' is also cheaper for the principal, because g' is the (unique) cost-optimizing contract for agent G during the last two periods. This is a contradiction.

[18] Note that we could weaken Condition 2.8: To ensure that the RHS of G is higher than the RHS of B it is sufficient to assume that the relative marginal probability of remaining unemployed $\frac{1}{\pi_i(z)} \frac{\partial \pi_i(z)}{\partial z}$ is higher for agent G than for agent B.

Second, suppose that the pooling contract p is identical to or steeper than the SW contract of agent G. The principal then offers p and a second contract b' that is identical to contract p with z^P_{T-1} and z^P_T substituted by z^B_{T-1} and z^B_T from the SW contract. Since the SW contract of agent B is flatter than the SW contract of agent G, as we have seen, we can infer the contradiction in the same way as in the first case. ■

Proof of Proposition 3.2

We restate the problem of offering unemployment insurance contracts to unemployed agents (ASUI):

$$\min_{\{z_1^b,\dots,z_T^b\},\{z_1^g,\dots,z_T^g\}} q[c(z_1^b) + \beta p_B(\hat{a}_1^B)[c(z_2^b) + \beta p_B(\hat{a}_2^B)[c(z_3^b) + \dots]\dots]] +$$
$$(1-q)[c(z_1^g) + \beta p_G(\hat{a}_1^G)[c(z_2^g) + \beta p_G(\hat{a}_2^G)[c(z_3^g) + \dots]\dots]]$$

subject to the *entitlement constraints* (EC)

$$V_1^{b,B} \geq \underline{V}, \tag{24}$$
$$V_1^{g,G} \geq \underline{V}, \tag{25}$$

the *adverse selection incentive constraints* (AS-IC)

$$V_1^{b,B} \geq V_1^{g,B}, \tag{26}$$
$$V_1^{g,G} \geq V_1^{b,G}. \tag{27}$$

and subject to the choice of effort by the agents.

We can now begin the proof of Proposition 3.2.

Proof. First, we give a formal definition of $\Gamma_t(V_t^{i,B}, V_t^{i,G})$. Recall that

$$V_t^{i,j} = z_t^i - \hat{a}_t^{i,j} + \beta(p_j(\hat{a}_t^{i,j})[z_{t+1}^i - \hat{a}_{t+1}^{i,j} + \beta(p_j(\hat{a}_{t+1}^{i,j})[\dots] + \dots)] + (1-p_j(\hat{a}_t^{i,j}))W_t), \tag{28}$$

where i denotes the type of contract, j the type of agent and $\hat{a}_t^{i,j}$ the choice of effort by agent j, given contract i at time t. Then the formal definition of Γ_t is straightforward:

$$\begin{aligned}\Gamma_t(V_t^{i,B}, V_t^{i,G}) \;=\; & \{(z_t^i, V_{t+1}^{i,B}, V_{t+1}^{i,G}) \mid \exists (z_t^i, \hat{z}_{t+1}^i, \dots, \hat{z}_T^i) s.t.\ j \in \{B, G\} \\ & V_t^{i,j} = z_t^i - \hat{a}_t^{i,j} + \beta(p_j(\hat{a}_t^{i,j})[\hat{z}_{t+1} \dots]\dots) \ \wedge \\ & V_t^{i,j} = z_t^i - \hat{a}_t^{i,j} + \beta(p_j(\hat{a}_t^{i,j})V_{t+1}^{i,j} + (1 - p_j(\hat{a}_t^{i,j})W_t)\}.\end{aligned}$$

In other words: The correspondence Γ_t maps a pair of state variables $(V_t^{i,B}, V_t^{i,G})$ in a given period t onto all triples $(z_t, V_{t+1}^{i,B}, V_{t+1}^{i,G})$ (where z_t denotes current utility and $V_{t+1}^{i,j}$ promised utilities) to which a contract $(z_t, ..., \hat{z}_T)$ exists that generates the corresponding lifetime utilities for agents B and G. Note that $\Gamma_t(V_t^{i,B}, V_t^{i,G})$ can be empty, because for some values of $(V_t^{i,B}, V_t^{i,G})$ of pairs of lifetime utilities of B and of G, there might be no sustaining contract.

Note also that the support of Γ_t is the "largest" set of pairs of lifetime utilities - all possible contracts are represented. Thus, adding the constraint

$$\Gamma_1(V^{b,B}, V^{b,G}) \neq \emptyset \qquad \Gamma_1(V^{g,B}, V^{g,G}) \neq \emptyset,$$

to ASUI does not change the problem - it simply means adding an empty constraint.

Therefore we can reformulate ASUI as follows:

$$\min_{(V^{b,B},V^{b,G},V^{g,B},V^{g,G})} \Bigg(\min_{\{z_1^b,...,z_T^b\},\{z_1^g,...,z_T^g\}} q[c(z_1^b) + \beta p_B(\hat{a}_1^{b,B})[c(z_2^b) + \beta p_B(\hat{a}_2^{b,B})[c(z_3^b) + ...]...]] + \\ (1-q)[c(z_1^g) + \beta p_G(\hat{a}_1^{g,G})[c(z_2^g) + \beta p_G(\hat{a}_2^{g,G})[c(z_3^g) + ...]...]] \Bigg)$$

subject to

$$\Gamma_1(V^{b,B}, V^{b,G}) \neq \emptyset \qquad \Gamma_1(V^{g,B}, V^{g,G}) \neq \emptyset, \tag{29}$$

subject to (EC)

$$V_1^{b,B} \geq \underline{V}, \tag{30}$$
$$V_1^{g,G} \geq \underline{V}, \tag{31}$$

and subject to (AS-IC)

$$V_1^{b,B} \geq V_1^{g,B}, \tag{32}$$
$$V_1^{g,G} \geq V_1^{b,G}. \tag{33}$$

The additional minimization over $(V^{b,B}, V^{b,G}, V^{g,B}, V^{g,G})$, together with the additional constraint (29) is empty since we allow for all pairs of utilities that correspond to a contract.

In this formulation we have dropped the implicit constraint on the choice of $(z_1^b, ..., z_T^b)$ and $(z_1^g, ..., z_T^g)$ by definition (28) of $V_1^{b,B}$, $V_1^{b,G}$, $V_1^{g,B}$ and $V_1^{g,G}$. As we have stated above, this is an empty constraint because we minimize over all pairs $(V_1^{b,B}, V_1^{b,G})$ and $(V_1^{g,B}, V_1^{g,G})$, to which a corresponding b and g exists.

We can decompose the objective function in the inner minimization problem into the sum of two separate minimization problems:

$$\min_{\{z_1^b,\ldots,z_T^b\}} q[c(z_1^b)+\beta p_B(\hat{a}_1^B)[c(z_2^b)+\beta p_B(\hat{a}_2^B)[c(z_3^b)+\ldots]\ldots]]+$$
$$\min_{\{z_1^g,\ldots,z_T^g\}}(1-q)[c(z_1^g)+\beta p_G(\hat{a}_1^G)[c(z_2^g)+\beta p_G(\hat{a}_2^G)[c(z_3^g)+\ldots]\ldots]]$$

This is because inside the brackets there is no interdependence of the two summands of the objective function or the implicit constraints (28).

Finally, we are left to show that the recursive formulation (16) of the contract problem solves the minimization problem

$$\min_{\{z_1^b,\ldots,z_T^b\}} c(z_1^b)+\beta p_B(\hat{a}_1^B)[c(z_2^b)+\beta p_B(\hat{a}_2^B)[c(z_3^b)+\ldots]\ldots] \tag{34}$$

subject to

$$\begin{aligned}
V^{b,B} &= z_1^b-\hat{a}_1^{b,B}+\beta(p_B(\hat{a}_1^{b,B})[z_2^b-\hat{a}_2^{b,B}+\beta(p_B(\hat{a}_2^{b,B})[\ldots]+\ldots)]+(1-p_B(\hat{a}_1^{b,B}))W_1)\\
V^{b,G} &= z_1^b-\hat{a}_1^{b,G}+\beta(p_G(\hat{a}_1^{b,G})[z_2^b-\hat{a}_2^{b,G}+\beta(p_G(\hat{a}_2^{b,G})[\ldots]+\ldots)]+(1-p_G(\hat{a}_1^{b,G}))W_1)
\end{aligned}$$

and the choice of effort by the agents CE (compare 2).

We prove the claim by induction over the number of periods T.

For $T=2$ we have to show that the following two formulations are equivalent:

$$\begin{aligned}
\min_{\{z_1^b,z_2^b\}} & \quad c(z_1^b)+\beta p_B(\hat{a}_1^B)c(z_2^b)\\
s.t. & \\
V^{b,B} &= z_1^b-\hat{a}_1^{b,B}+\beta(p_B(\hat{a}_1^{b,B})z_2^b+(1-p_B(\hat{a}_1^{b,B}))u(w))\\
V^{b,G} &= z_1^b-\hat{a}_1^{b,G}+\beta(p_G(\hat{a}_1^{b,G})z_2^b+(1-p_G(\hat{a}_1^{b,G}))u(w))\\
CE &
\end{aligned}$$

and

$$\begin{aligned}
C_1^B(V^{b,B},V^{b,G}) &= \min_{\{z_1,V_2^{b,B},V_2^{b,G}\}\in\Gamma_1(V^{b,B},V^{b,G})} c(z_t)+\beta p_B(a^B)C_2^B(V_2^{b,B},V_2^{b,G})\\
s.t. & \\
V^{b,B} &= z_1-\hat{a}_1^{b,B}+\beta[p_B(\hat{a}_1^{b,B})V_2^{b,B}+(1-p_B(\hat{a}_1^{b,B}))u(w)]\\
V^{b,G} &= z_1-\hat{a}_1^{b,G}+\beta[p_G(\hat{a}_1^{b,G})V_2^{b,G}+(1-p_G(\hat{a}_1^{b,G}))u(w)]\\
CE & \\
V_2^{b,B} &= V_2^{b,G}=z_2^b.
\end{aligned}$$

Recall that the last constraint is due to the death of the agents at the end of period $T = 2$. Substituting z_2^b for $V_2^{b,B}$ and $V_2^{b,G}$ and $c(z_2^b)$ for $C_2^B(V_2^{b,B}, V_2^{b,G})$ delivers the equivalence.

Now assume the claim holds for $T-1$. Then the problem 34 for T periods can be rewritten as

$$
\begin{array}{rcl}
\min\limits_{\{z_1^b\}} & & c(z_1^b) + \beta p_B(\hat{a}_1^B)\Big(\min\limits_{\{z_2^b,\dots,z_T^b\}} c(z_2^b) + \beta p_B(\hat{a}_2^B)[c(z_3^b) + \dots]\dots\Big) \\
s.t. & & \\
V^{b,B} & = & z_1^b - \hat{a}_1^{b,B} + \beta(p_B(\hat{a}_1^{b,B})V_2^{b,B} + (1 - p_B(\hat{a}_1^{b,B}))W_1) \\
V^{b,G} & = & z_1^b - \hat{a}_1^{b,G} + \beta(p_G(\hat{a}_1^{b,G})V_2^{b,G} + (1 - p_G(\hat{a}_1^{b,G}))W_1) \\
CE & &
\end{array}
$$

Note that all we have done is:

1. to split up the summation in the objective function
2. to separate the choice variable z_1^b from the other choice variables $z_2^b, \dots, z_T^b$
3. to reformulate the constraints in terms of the lifetime utilities $V_t^{i,j}$ as defined by equation 28.

By using the induction hypothesis we may identify the variables $V_2^{b,B}$ and $V_2^{b,G}$ with the state variables and the term in the brackets with the recursive cost function $C_2(V_2^{b,B}, V_2^{b,G})$ of the $T-1$ version of the problem. Limiting the choice of $(z_1^b, V_2^{b,B}, V_2^{b,G})$ to elements of $\Gamma_1(V^{b,B}, V^{b,G})$ ensures by construction of Γ that the former correspond to feasible contracts $(z_1^b, z_2^b, \dots, z_T^b)$. This effectively proves the theorem. ■

Proof of Proposition 3.3

Proof. In order to simplify the proof we introduce a normalization: The utility from consuming the wage w is set to zero. Thus, all W_t become zero too, and the entitlement utilities of the unemployed agents take non-positive values. Note that the lower bounds for the entitlements, stemming from the lower bound on the benefit utility $\underline{z}$, thus shift downward each period along the backward induction.

First, we look at the agents' problem. Recall it takes the form

$$V_t^i = \max_a z_t - a + \beta[p_i(a)V_{t+1}^i + (1 - p_i(a))W_{t+1}].$$

Given our normalization, we obtain the following first order condition at an interior solution:

$$p_i'(a_t^i) = \frac{1}{\beta V_{t+1}^i} \tag{35}$$

By using the Inada condition in Condition 2.4 we ensure that the interior solution always applies.

The case of $\mathbf{t = T - 1}$: We start with the case of $\Gamma_t(V_t^B, V_t^G)$ with $t = T - 1$. Mathematically speaking, the next-to-last period is different from the previous ones in that there is an additional constraint on the choice variables V_T^i: the boundary conditions 19, 20, namely $V_T^B = V_T^G = z_T$. This is the very reason why, given the pair of state variables (V_{T-1}^B, V_{T-1}^G), there is only one choice left for the principal. First, let us look at the Law of Motion (LOM) for the state variables V_{T-1}^B and V_{T-1}^G:

$$\begin{aligned} z_{T-1} - a_{T-1}^B + \beta p_B(a_{T-1}^B)V_T^B &= V_{T-1}^B, \\ z_{T-1} - a_{T-1}^G + \beta p_G(a_{T-1}^G)V_T^G &= V_{T-1}^G, \end{aligned}$$

where we will drop the time index from the effort variables a_{T-1}^i. In the following, we will denote the difference between the entitlements of the agents by:

$$\Delta_t := V_t^G - V_t^B. \tag{36}$$

With this new notation, and remembering both our normalization and $V_T^i = z_T$, we solve the LOM for z_{T-1}, equalize both equations and solve for Δ_{T-1}:

$$\Delta_{T-1} = a^B - a^G + \beta p_G(a^G)z_T - \beta p_B(a^B)z_T \tag{37}$$

We want to further simplify equation 37. In the next-to-last period, the first order condition of the agents' problem (35) takes the following form

$$p_B'(a^B) = p_G'(a^G) = \frac{1}{\beta z_T}. \tag{38}$$

Again by using Condition 2.4, the p_i' are strictly increasing functions

$$p_i' : (0, \infty) \longrightarrow (-\infty, 0).$$

Remark A.1 *Given Condition 2.3, the principal will never promise an entitlement above W_t ($= 0$ under our normalization), since at W_t the agents stop searching (i.e. $a_t^i = 0$) and their probability of remaining unemployed becomes $p_i(a_t^i) = 1$.*

Thus in particular $V_T^i = z_T < 0$.

From this we deduce that the p_i' are one-to-one and onto. Therefore the following function $\gamma(a^G)$ is well-defined:

$$\gamma(a^G) := (p_B')^{-1} \circ p_G'(a^G).$$

Now we have everything at hand to define Δ_{T-1} as a function of a^G:

$$\Delta_{T-1}(a^G) = \gamma(a^G) - a^G + \frac{p_G(a^G) - p_B(\gamma(a^G))}{p'_G(a^G)} \tag{39}$$

In order to show point 3 of Proposition 3.3, we have to demonstrate that $\Delta_{T-1}(.)$ is invertible.

We do so by proving:

$$\Delta'_{T-1}(a^G) > 0. \tag{40}$$

Using the agents' first order condition 35 and

$$\gamma'(a^G) = \frac{p''_G(a^G)}{p''_B(\gamma(a^G))}$$

we calculate

$$\Delta'_{T-1}(a^G) = [p_B(\gamma(a^G)) - p_G(a^G)]\frac{p''_G(a^G)}{(p'_G(a^G))^2}. \tag{41}$$

By Condition 2.4 we know that $p''_G(.) > 0$, and since $p_B(\gamma(a^G)) > p_G(a^G)$ by Condition 2.6, assertion 40 follows.

Finally we observe that, again by Condition 2.4:

$$\lim_{a^G \to 0} \Delta_{T-1}(a^G) = 0. \tag{42}$$

Together with 40 we deduce that as agent B's entitlement V^B_{T-1} approaches agent G's one V^G_{T-1}, the effort of the agent G a^G (as well as the effort of agent B) goes to zero. Because of 35 this means that the benefit for the last period z_T has to converge to zero, i.e. the wage consumption utility.

Summarizing our results so far, we can state the following: Given entitlements V^B_{T-1} and V^G_{T-1} such that $\Delta_{T-1} \geq 0$, we can find a unique corresponding choice of effort by agent G a^G (for the time being, we neglect the lower bound $\underline{z}$ on the benefits z_t). From this we can calculate - uniquely - the choice of effort by agent B a^B and the benefit for the last period z_T from equation 35, and the benefit of the next to last period z_{T-1} from LOM. All these functions are differentiable. As Δ_{T-1} goes to zero, the benefit of the last period z_T goes to zero, i.e. the cost of the benefit converges to that of the wage.

We finally have to look at the set of feasible entitlements V^B_{T-1} and V^G_{T-1}. If $\underline{z} = -\infty$, so z can take any value, we infer from 42 that the upper bound $\overline{V}^B_t(.)$ on V^B_t, given V^G_t, is

$$\overline{V}^B_t(V^G_t) = V^G_t.$$

As for the lower bound, we calculate

$$\underline{V}_t^B(V_t^G) = \lim_{a^G \to \infty} V_t^G - \Delta_{T-1}(a^G).$$

Now let $\underline{z} > -\infty$. Then there is a natural lower bound $\underline{V}_{T-1}^G$, namely the stochastically discounted sum of the bounds on z_{T-1} and z_T. Given $V_{T-1}^G \in [\underline{V}_{T-1}^G, 0]$ we now have to prove that there is a lower and an upper bound $\underline{V}_{T-1}^B(V_{T-1}^G)$ and $\overline{V}_{T-1}^B(V_{T-1}^G)$ on the corresponding feasible V_{T-1}^B. Because of 35, the lower bound on z_T translates into an upper bound $\overline{a}^G$ on the corresponding choices of effort of agent G. It is attained with equality. By 36 and 40 we find the lower bound

$$\underline{V}_{T-1}^B(V_{T-1}^G) = V_{T-1}^G - \Delta_{T-1}(\overline{a}^G).$$

As for the upper bound $\overline{V}_{T-1}^B(V_{T-1}^G)$, one can see intuitively that V_{T-1}^B is bounded by V_{T-1}^G (for a rigorous argument, see point 1 in the proof of 3.7). However, V_{T-1}^B does not necessarily attain this bound because of an additional constraint: $z_{T-1} \geq \underline{z}$. From the LOM and 35 we know

$$z_{T-1} = V_{T-1}^G + a^G - \frac{p_i(a^G)}{p_i'(a^G)}$$

The RHS is increasing in a^G, so a lower bound on z_{T-1} implies a lower bound on the effort of the second type, $\underline{a}^G$ (note that because of our normalization, the reference points for each period have been shifted downwards). Because of 40, a lower bound on Δ_{T-1} ensues. Given V_{T-1}^G, we thus find the upper bound on V_{T-1}^B:

$$\overline{V}_{T-1}^B(V_{T-1}^G) = V_{T-1}^G - \Delta_{T-1}(\underline{a}^G).$$

We see that V_{T-1}^B attains V_{T-1}^G only if the lower bound $\underline{a}^G$ becomes zero (the smallest possible effort). Since $\Delta_{T-1}(.)$ is an increasing function, we see that all values $V_{T-1}^B \in [\underline{V}_{T-1}^B, \overline{V}_{t-1}^B]$ are attainable as long as $\overline{a}^G > \underline{a}^G$. This must be the case for $V_{T-1}^G \geq \underline{V}_{T-1}^G$, since then there are corresponding benefit values z_{T-1}, z_T such that $z_i \geq \underline{z}$. Finally, because of the Theorem of the Maximum, both $\underline{a}_{T-1}$ and $\overline{a}_{T-1}$ depend continuously on V_{T-1}^G, and since Δ_{T-1} is a smooth function, the lower and the upper bound $\underline{V}_{T-1}^B$ and $\overline{V}_{T-1}^B$ are continuous functions of V_{T-1}^G.

So for period $T-1$, we have shown that the set of feasible values takes the form stated in the theorem. Note in particular that this set is compact and connected.

The case of $\mathbf{t \leq T-2}$: In this subsection, we prove assertion 1. As stated above, the crucial difference between the next-to-last period and the previous ones is the boundary condition of the last periods 19 and 20. Before, for every feasible pair of state variables (V_{T-1}^B, V_{T-1}^G) - or more precisely, for the difference of these state variables Δ_{T-1} - one corresponding choice of effort a_{T-1}^G existed that determined all choice variables (z_{T-1}, V_T^B, V_T^G). As we will see below, for each pair of feasible state variables (V_{t-1}^B, V_{t-1}^G) (again, more precisely, to the difference of these state variables Δ_{t-1}) there is a line of possible choices of effort a_{t-1}^G that parameterizes a compact and connected path of choice variables $(z_{t-1}(.), V_t^B(.), V_t^G(.))$.

We have a look at the LOM once more. With the help of the agents' first order condition we transform it into

$$\begin{aligned} z_t - a^B + \frac{p_B(a^B)}{p_B'(a^B)} &= V_t^B, \\ z_t - a^G + \frac{p_G(a^G)}{p_G'(a^G)} &= V_t^G, \end{aligned}$$

where again we have dropped the time index from a_t^i. This inspires the definition of the following functions $(i = 1, 2)$

$$f_i(a^i) = a^i - \frac{p_i(a^i)}{p_i'(a^i)}.$$

From the LOM we can now derive a necessary equation for the choice variables (as represented by the a^is, replacing the V_t^is) to hold:

$$\Delta_t + f_G(a^G) = f_B(a^B), \tag{43}$$

where we have used definition 36.

Let us have a closer look now at f_i. From

$$f_i' = \frac{p_i p_i''}{(p_i')^2} > 0 \tag{44}$$

we can see that it is a strictly increasing function (bearing in mind Condition 2.4). Moreover, we calculate

$$\lim_{a^i \to 0} f_i(a^i) = 0, \tag{45}$$

$$\lim_{a^i \to \infty} f_i(a^i) = \infty. \tag{46}$$

Now note that there is a natural lower bound $\underline{V}_t^G$ on each V_t^G, namely the stochastically discounted sum of the $z_{\hat{t}}$s (where $\hat{t} = t, ..., T$). In the case of $T-1$, we have shown that the set of jointly feasible values V_{T-1}^B, V_{T-1}^G takes the form stated in the theorem. So let $\Gamma_t(V_t^B, V_t^G)$ be non-empty and take the form of a path in the space

$(z_t, V_{t+1}^B, V_{t+1}^G)$ for $V_t^B \in [\underline{V}_t^B(V_t^G), \overline{V}_t^B(V_t^G)]$ with $V_t^G \geq \underline{V}_t^G$. We have to show first that $\Gamma_{t-1}(V_{t-1}^B, V_{t-1}^G)$ is then non-empty for $V_{t-1}^B \in [\underline{V}_{t-1}^B(V_t^G), \overline{V}_{t-1}^B(V_t^G)]$ for some continuous functions $\underline{V}_{t-1}^B$, $\overline{V}_{t-1}^B$ when $V_{t-1}^G \geq \underline{V}_{t-1}^G$, and takes the form of a path in (z_{t-1}, V_t^B, V_t^G).

Put differently, we have to ask for which pairs (V_{t-1}^B, V_{t-1}^G) are there choice variables (z_{t-1}, V_t^B, V_t^G) that are jointly feasible. By the agents' first order condition (35) we can replace V_t^B and V_t^G by the corresponding choices of effort a_{t-1}^B and a_{t-1}^G (we will drop the time index in the sequel). The effort choices a^B and a^G have to satisfy equation 43. Since $\Delta_{t-1} \geq 0$ and by 44, 45 and 46, for all $a^G \geq 0$ we can find a corresponding $a^B \geq 0$. By the LOM, we can furthermore determine z_{t-1} once a^G is given. Thus the number of choice variables $(z_{t-1}(a^G), V_t^B(a^G), V_t^G(a^G))$ is reduced to the "choice" variable a^G. All functions are combinations of differentiable functions and thus differentiable. The curve $\phi_{\Delta_{t-1}}$, which is parameterized in a^G, is defined as the projection of the triple of choice variables into the two-dimensional space $(V_t^B(a^G), V_t^G(a^G))$.

We have reduced the choice problem to one variable, but in which a^G corresponds to *feasible* triples (z_{t-1}, V_t^B, V_t^G)? First we look at the constraint $z_{t-1} \geq \underline{z}$. As in the preceding subsection, by using the LOM

$$z_{t-1} = V_{t-1}^G + f_G(a^G)$$

it translates into a constraint[19]

$$a^G \geq \underline{a}^G = \begin{cases} f_G^{-1}(\underline{z} - V_{t-1}^G) & : \quad V_{t-1}^G \leq \underline{z} \\ 0 & : \quad V_{t-1}^G > \underline{z} \end{cases} \tag{47}$$

Second we have to ask: Which of the pairs of entitlements $(V_t^B(a^G), V_t^G(a^G))$ are feasible? The answer is, those for which $\Gamma_t(V_t^B, V_t^G)$ is non-empty. In other words: Given V_{t-1}^B and V_{t-1}^G, the set of feasible choices is the intersection of the curve $\phi_{\Delta_{t-1}}$ defined by 43, parameterized in a^G with $a^G \geq \underline{a}^G$, and the set of (V_t^B, V_t^G) with $\Gamma_t(V_t^B, V_t^G) \neq \emptyset$. Figure 1 depicts the intersection for the case of period 4 of 12 in an example from our simulation. The solid lines represent the bounds $\underline{V}_4^B(V_4^G)$ and $\overline{V}_4^B(V_4^G)$, while the dotted and the dashed line are curves ϕ_{Δ_3} with two different values for Δ_3.

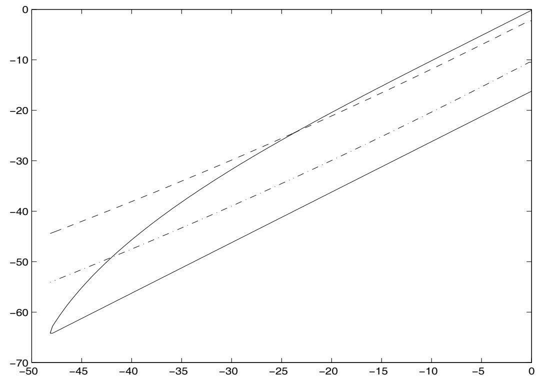
Two things remain to be shown:

1. We have to show that the set of (V_{t-1}^B, V_{t-1}^G), for which the intersection is non-empty, itself takes the form of a set bounded by functions $\underline{V}_{t-1}^B$ and $\overline{V}_{t-1}^B$.

2. We have to show that if curve 43 intersects the set of feasible values (V_t^B, V_t^G),

[19] If $\underline{z} = -\infty$, by our definition there is no limit on a^G.

Figure 1: Set of Jointly Feasible Entitlements in Period 4 of 12



x-axis: Entitlements of type B, y-axis: Entitlements of type G
with projection of sets of agents' choice sets

it cuts the bounds at most twice, so that the set of feasible choices is connected.

To show the *first assertion*, we look more closely at the family of curves

$$\phi_{\Delta_{t-1}} : a^G \longrightarrow [\phi_B(a^G), \phi_G(a^G)]_{\Delta_{t-1}},$$

where

$$\phi_B(a^G) = \frac{1}{\beta p'_B(f_B^{-1}(\Delta_{t-1} + f_G(a^G)))}, \tag{48}$$

$$\phi_G(a^G) = \frac{1}{\beta p'_G(a^G)}. \tag{49}$$

Since ϕ_G is one-to-one, the curves can also be understood as a function

$$V_t^B = \phi_{\Delta_{t-1}}(V_t^G).$$

We now want to prove the following. The curves are "decreasing" in Δ_{t-1}, i.e.

$$\Delta_{t-1} < \Delta^*_{t-1} \Rightarrow \phi_{\Delta_{t-1}}(V_t^G) > \phi_{\Delta^*_{t-1}}(V_t^G). \tag{50}$$

We do so by calculating the derivative

$$\partial_{\Delta_{t-1}}(\phi_{\Delta_{t-1}})(V^G) = -\frac{1}{(\beta p'_1(a^B))^2} \cdot \beta p''_1(a^B) \cdot \frac{1}{f'_{\theta_1}(\Delta_{t-1} + f_{\theta_2}(a^G))} < 0,$$

which is negative because of 44 and Condition 2.4. The property of $\phi_{\Delta_{t-1}}$ is reflected by its dotted line and the dashed representation in Figure 1.

By the induction hypothesis, the set of (V_t^B, V_t^G), for which $\Gamma_t(V_t^B, V_t^G) \neq \emptyset$, is compact and connected. Thus we deduce from 50 that there are $\overline{\Delta}_{t-1}$ and $\underline{\Delta}_{t-1}$ so that the curves $\phi_{\Delta_{t-1}}$ intersect the set for $\underline{\Delta}_{t-1} \leq \Delta_{t-1} \leq \overline{\Delta}_{t-1}$ and do not intersect for $\Delta_{t-1} < \underline{\Delta}_{t-1}$ and $\Delta_{t-1} > \overline{\Delta}_{t-1}$ (of course $\underline{\Delta}_{t-1}$ could be smaller than zero, the lower limit for Δ_{t-1}). From this ensues the existence of two functions, $\underline{V}_{t-1}^B(V_{t-1}^G)$ and $\overline{V}_{t-1}^B(V_{t-1}^G)$, limiting the set of feasible pairs (V_{t-1}^B, V_{t-1}^G).

To show the *second assertion*, we have to look more closely at the shape of the curve $\phi_{\Delta_{t-1}}$ as well as the limiting functions $\underline{V}_{t-1}^B(.)$ and $\overline{V}_{t-1}^B(.)$. First, we prove that the derivative of $\phi_{\Delta_{t-1}}$ is *smaller than one*. We do so by showing that

$$D(.) \circ \phi_G^{-1}(V_{t-1}^G) := (\phi_G(.) - \phi_B(.)) \circ \phi_G^{-1}(V_{t-1}^G)$$

is increasing in V_{t-1}^G, i.e. the derivative of $\phi_{\Delta_{t-1}}$ is below the one of the diagonal:

$$\partial_{V_{t-1}^G} D(\phi_G^{-1}(V_{t-1}^G)) = 1 - \partial_{V_{t-1}^G} \phi_B(\phi_G^{-1}(V_{t-1}^G)) > 0.$$

Since we know that

$$\partial_{V_{t-1}^G}(\phi_G^{-1})(V_{t-1}^G) < 0$$

from (35), it is sufficient to show that

$$D'(a^G) < 0.$$

Using $a^B := f_B^{-1}(\Delta_{t-1} + f_G(a^G))$ we calculate

$$\begin{aligned}
D'(a^G) &= -\frac{p_G''(a^G)}{\beta(p_G'(a^G))^2} + \frac{p_B''(a^B)}{\beta(p_B'(a^B))^2} \cdot \frac{f_G'(a^G)}{f_B'(a^B)} \\
&= -\frac{p_G''(a^G)}{\beta(p_G'(a^G))^2} + \frac{p_B''(a^B)}{\beta(p_B'(a^B))^2} \cdot \frac{p_G(a^G)p_G''(a^G)}{(p_G'(a^G))^2} \cdot \frac{(p_B'(a^B))^2}{p_B(a^B)p_B''(a^B)} \\
&= \left(\frac{p_G(a^G)}{p_B(a^B)} - 1\right) \cdot \frac{p_G''(a^G)}{\beta(p_G'(a^G))^2}.
\end{aligned}$$

The last expression is negative by Conditions 2.4 and 2.6. Now, the second assertion follows if we can show that the derivative of the boundary functions $\underline{V}^B(.)$ and $\overline{V}^B(.)$ is *greater than one*, for then $\phi_{\Delta_{t-1}}$ crosses them at most once. So by the induction hypothesis, assume that $\underline{V}_t^B(.)$ and $\overline{V}_t^B(.)$ have a derivative greater or equal than one (note that this is certainly true for the case of $t = T - 1$).

According to what we have shown above, $\overline{\Delta}_{t-1}$ and $\underline{\Delta}_{t-1}$ limit the set of values $\Delta_{t-1} = V_{t-1}^G - V_{t-1}^B$ for which $\phi_{\Delta_{t-1}}$ intersects the set of feasible (V_t^B, V_t^G). From this we might be tempted to deduce immediately both $\underline{V}_{t-1}^{B}$ and $\overline{V}_{t-1}^{B}$ must be linear functions with derivative one, for apparently the limits only depend on the difference $\Delta_{t-1} = V_{t-1}^{G} - V_{t-1}^{B}$. Note, however, that the starting point $\underline{a}^{G}$ (see equation 47) for each curve $\phi_{\Delta_{t-1}}$ is shifting upwards as V_{t-1}^{G} is falling. Thus, since by induction hypothesis $\underline{V}_{t}^{B}$ and $\overline{V}_{t}^{B}$ are increasing more steeply than the $\phi_{\Delta_{t-1}}$, we may deduce that

1. $\underline{V}_{t-1}^{B}(.)$ is indeed linear with derivative one because the $\phi_{\Delta_{t-1}}$s cross the function $\underline{V}_{t}^{B}(.)$ at the lower bound $\underline{V}_{t}^{G}$ at a high value for a^{G}.

2. For lower values of V_{t-1}^{G}, the smallest Δ_{t-1} for which $\phi_{\Delta_{t-1}}$ intersects the set of feasible values (V_{t}^{B}, V_{t}^{G}) is below the one that would have been obtained with $\underline{a}^{G}$ fixed. Since the latter would have corresponded to a linear upper bound $\overline{V}_{t}^{B}(.)$ with derivative one, we conclude that $\overline{V}_{t}^{B}(.)$ has to rise more steeply than this, i.e. that its derivative is greater than one.

Thus by induction, we have shown that $\phi_{\Delta_{t-1}}$ and $\underline{V}_{t-1}^{B}(.)$ and $\overline{V}_{t-1}^{B}(.)$ cross only once and the second assertion about the form of the correspondence Γ_{t-1} ensues. This concludes the proof of Proposition 3.3. ■

Proof of Proposition 3.7

Proof. To prove this proposition, we have to show that at the solution:

1. The entitlement constraint 15 of type G is slack;

2. The entitlement constraint 14 of type B is binding; and

3. The incentive constraint 13 of type G is binding.

Beginning with 1 we show that for all contracts, $V^{G} > V^{B}$. The assertion then follows by $V^{b,G} > V^{b,B}$ and agent B's entitlement constraint 14.

So we consider a feasible UI contract. Given any set of effort choices $(a_{1}^{B}, a_{2}^{B}, ..., a_{T-1}^{B})$ of agent B, the same set of choices would yield a higher value of total expected lifetime utility for agent G than for agent B, $V^{G}(\vec{a}^{B}) > V^{B}(\vec{a}^{B})$. This is the case because firstly (total) utility when employed is higher than (total) utility when unemployed (compare the remarks 3.4 and A.1), and secondly by condition 2.6, first part, $p_{B}(a^{B}) > p_{G}(a^{B})$ for any $a^{B} > 0$. Thus, in particular, at the optimum $V^{G} > V^{B}$.

We now prove point 2 by contraction. Suppose that for the solution contracts b, $(z_{1}^{b}, ..., z_{T}^{b})$, and g, $(z_{1}^{g}, ..., z_{T}^{g})$, the constraint 14 did not bind. For sufficiently high $\underline{V}$ we may assume that all $z_{t}^{i} > \underline{z}$ for all t, in particular for $t = 1$. But then create new contracts b' and g' by replacing z_{1}^{i} by $z_{1}^{i} - \epsilon$ $(i = b, g)$ for some $\epsilon > 0$

with $z_1^i - \epsilon > \underline{z}$. These contracts are certainly feasible. They are also incentive compatible, since the entitlements $V_1^{i,j}$ are reduced by the same amount. However, the new contracts b' and g' are less costly for the principal, since the cost function $c(.)$ is strictly increasing. This is a contradiction.

Point 3 is proved by an argument simular to the one in point 2. ■

Proof of Corollary 3.8

Proof. The first assertion is point 2 in the proof of 3.7. To see the second assertion, note that, given that $V^{g,B}$ is chosen optimally for each value of $V^{g,G}$, the cost function of contract g strictly increases in $V^{g,G}$. Moreover, in a full information optimum (i.e. the pure moral hazard case for both contracts), the optimal $V^{b,G}$ (optimal with respect to $V^{b,B} = \underline{V}$) can be characterized by a first order condition. We thus obtain a first order reduction of costs for contract g by lowering $V^{g,G} = V^{b,G}$ (constraint 13 is binding) below the value of $V^{b,G}$ in a full information optimum, whereas there is only a second order increase in costs for contract b. ■

Proof of Corollary 3.9

Proof. The fact that $V^{g,G} > \underline{V}$ has been proved in Proposition 3.7; we can therefore look at the second assertion. In our framework, we can recover the SW contracts (i.e. the contracts from the pure moral hazard environment) at a given level of entitlement $V^{i,i}$ by solving ($i \neq j$):

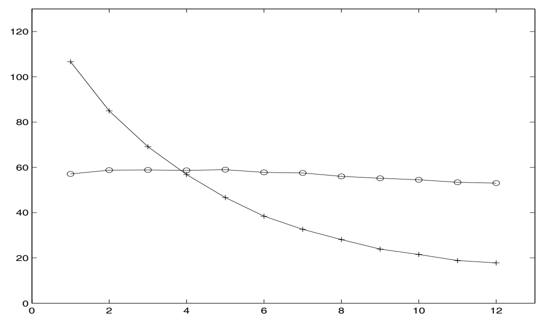
$$\begin{aligned} &\min_{V^{i,j}} C_1^i(V^{i,i}, V^{i,j}) \\ s.t. \quad &LOM, MH - IC \end{aligned}$$

and applying forward induction afterwards. This is because by minimizing the costs of contract i with respect to its value for agent j, we simply neglect the impact of this value for the optimal contract.

Now, if our objective function is optimized without further restriction, we recover the optimal contract from the pure moral hazard environment, because the value $V^{g,B}$ of contract g for type B does not appear in the cost function of contract b. ■

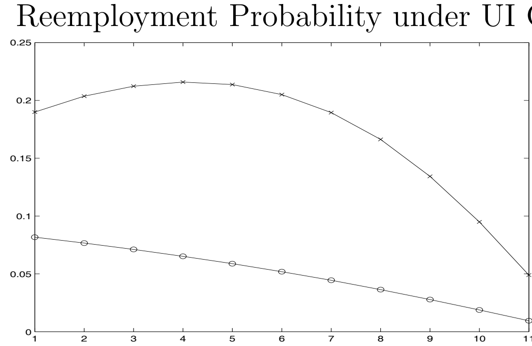
B Figures

Figure 2: UI Contracts with Adverse Selection



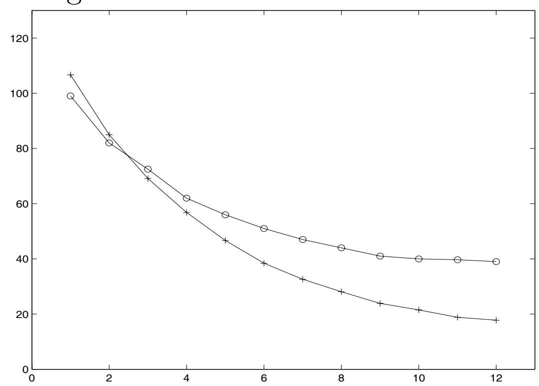
y-axis: Replacement rate, x-axis: Period
'o' - contract b, '+' - contract g
Entitlement bound $\underline{V} = -25$

Figure 3: Reemployment Probability under UI Contracts



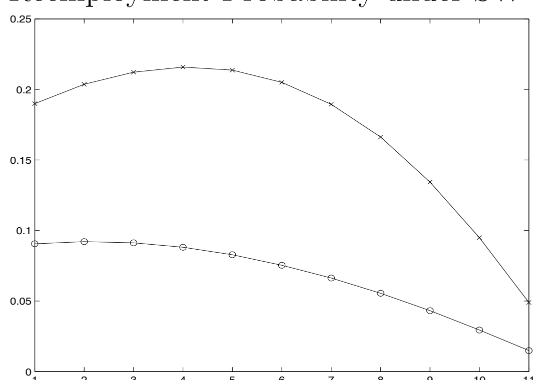
y-axis: Reemployment probability, x-axis: Entitlement of type G
'o' - contract b, '+' - contract g
Entitlement bound $\underline{V} = -25$

Figure 4: Shavell-Weiss Contracts



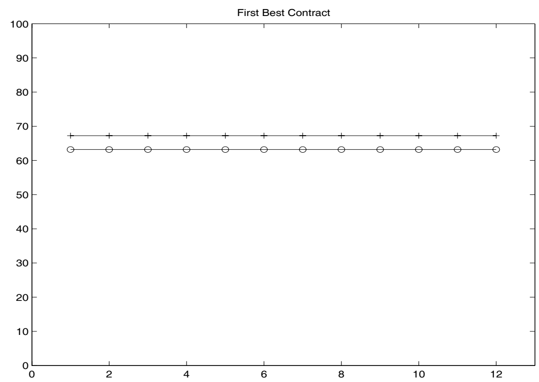
y-axis: Replacement rate, x-axis: Period
'o' - contract b, '+' - contract g
Entitlement bound $\underline{V} = -25$

Figure 5: Reemployment Probability under SW Contracts



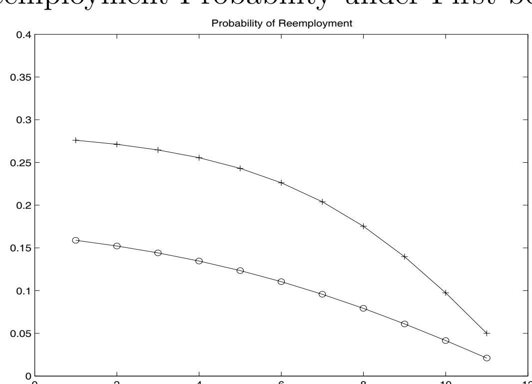
y-axis: Reemployment probability, x-axis: Period
'o' - contract b, '+' - contract g
Entitlement bound $\underline{V} = -25$

Figure 6: First-best Contracts



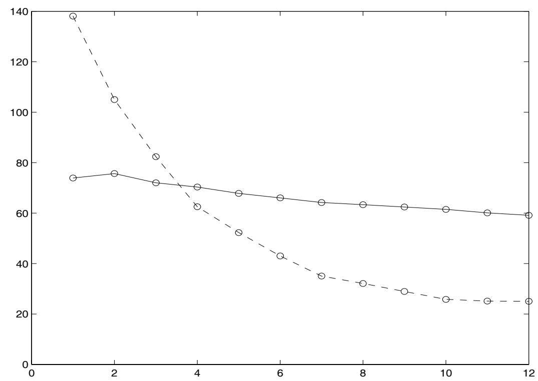
y-axis: Replacement rate, x-axis: Period
'o' - contract b, '+' - contract g
Entitlement bound $\underline{V} = -25$

Figure 7: Reemployment Probability under First-best Contracts



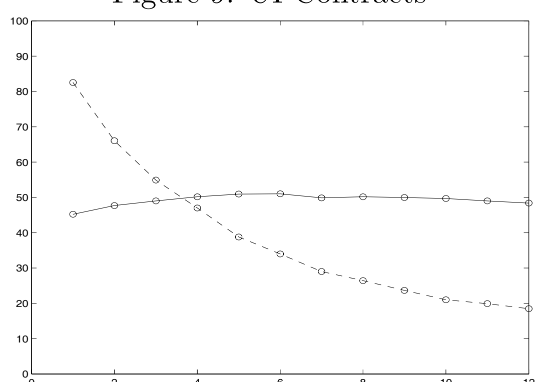
y-axis: Reemployment probability, x-axis: Period
'o' - contract b, '+' - contract g
Entitlement bound $\underline{V} = -25$

Figure 8: UI Contracts



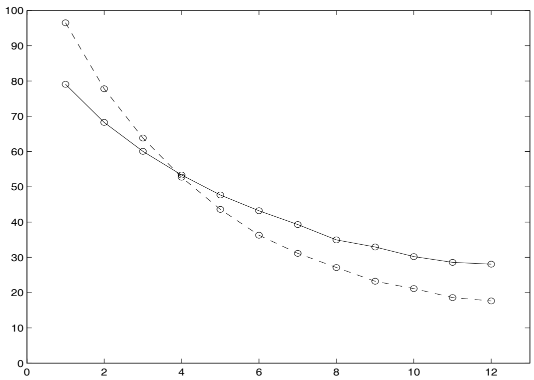
y-axis: Replacement rate, x-axis: Period
Entitlement bound $\underline{V} = -20$
$\cong$ Certainty equivalent of 68.67% of the wage per period

Figure 9: UI Contracts



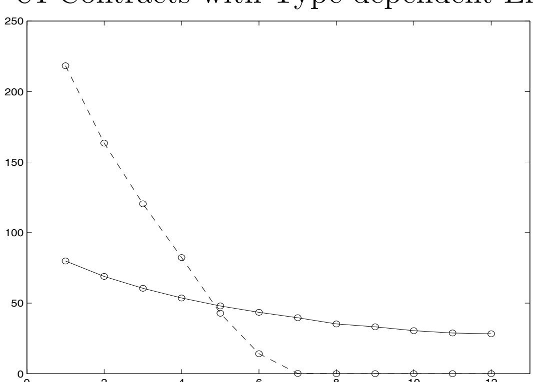
y-axis: Replacement rate, x-axis: Period
Entitlement bound $\underline{V} = -30$
$\cong$ Certainty equivalent of 55.21% of the wage per period

Figure 10: UI Contracts with Type-dependent Entitlements



y-axis: Replacement rate, x-axis: Period
Entitlement bound $\underline{V}_B = -30$, $\underline{V}_G = -22$
$\cong$ Certainty equivalent of 55.21% resp. 65.86% of the wage per period

Figure 11: UI Contracts with Type-dependent Entitlements



y-axis: Replacement rate, x-axis: Period
Entitlement bound $\underline{V}_B = -30$, $\underline{V}_G = -15$
$\cong$ Certainty equivalent of 55.21% resp. 75.96% of the wage per period